1. Multi-planetary systems
2. Saturn's Rings
3. The collisional N-body code REBOUND

Hanno Rein @ Northwestern, March 2012

Migration in a non-turbulent disc

Planet formation

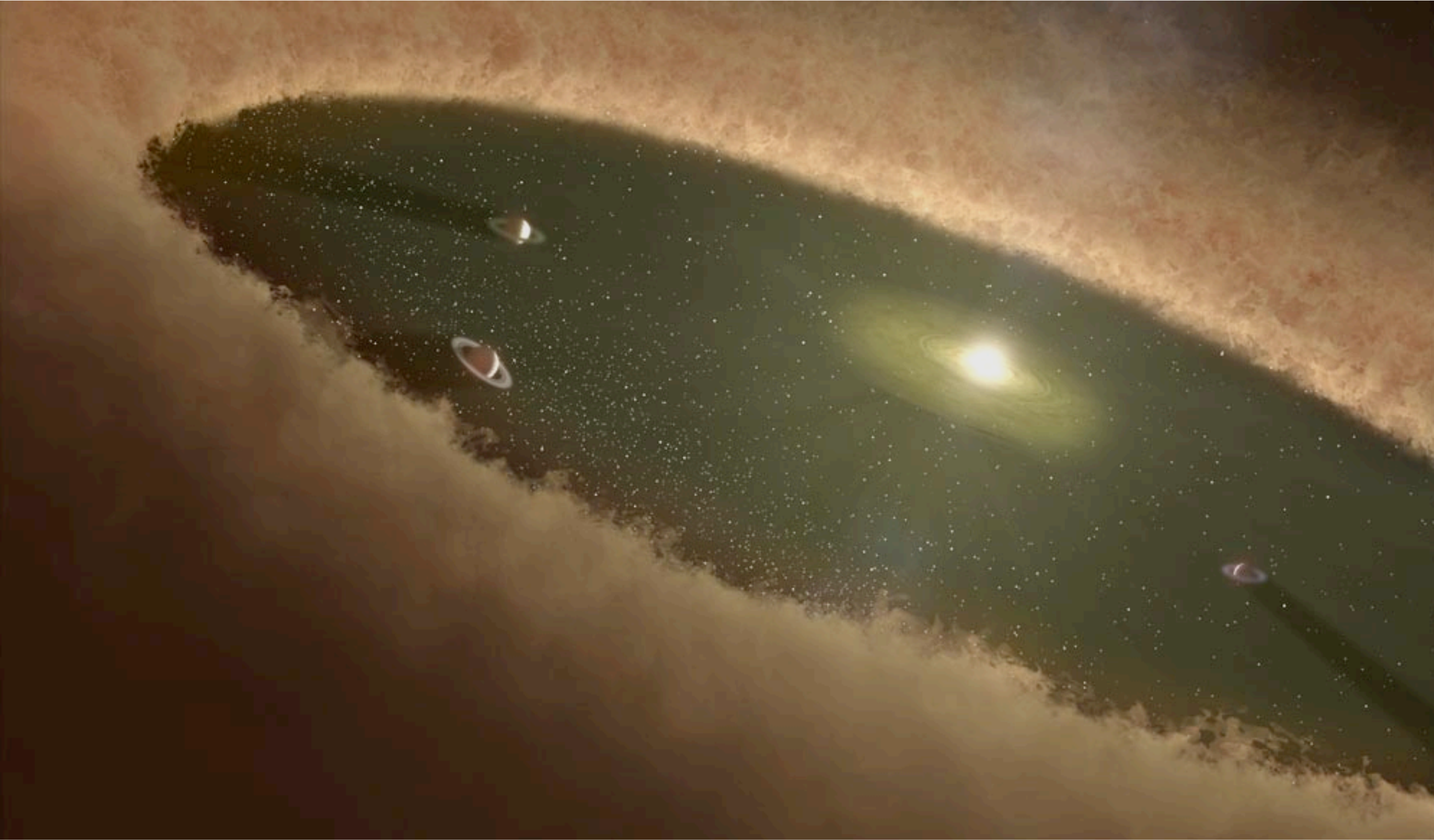


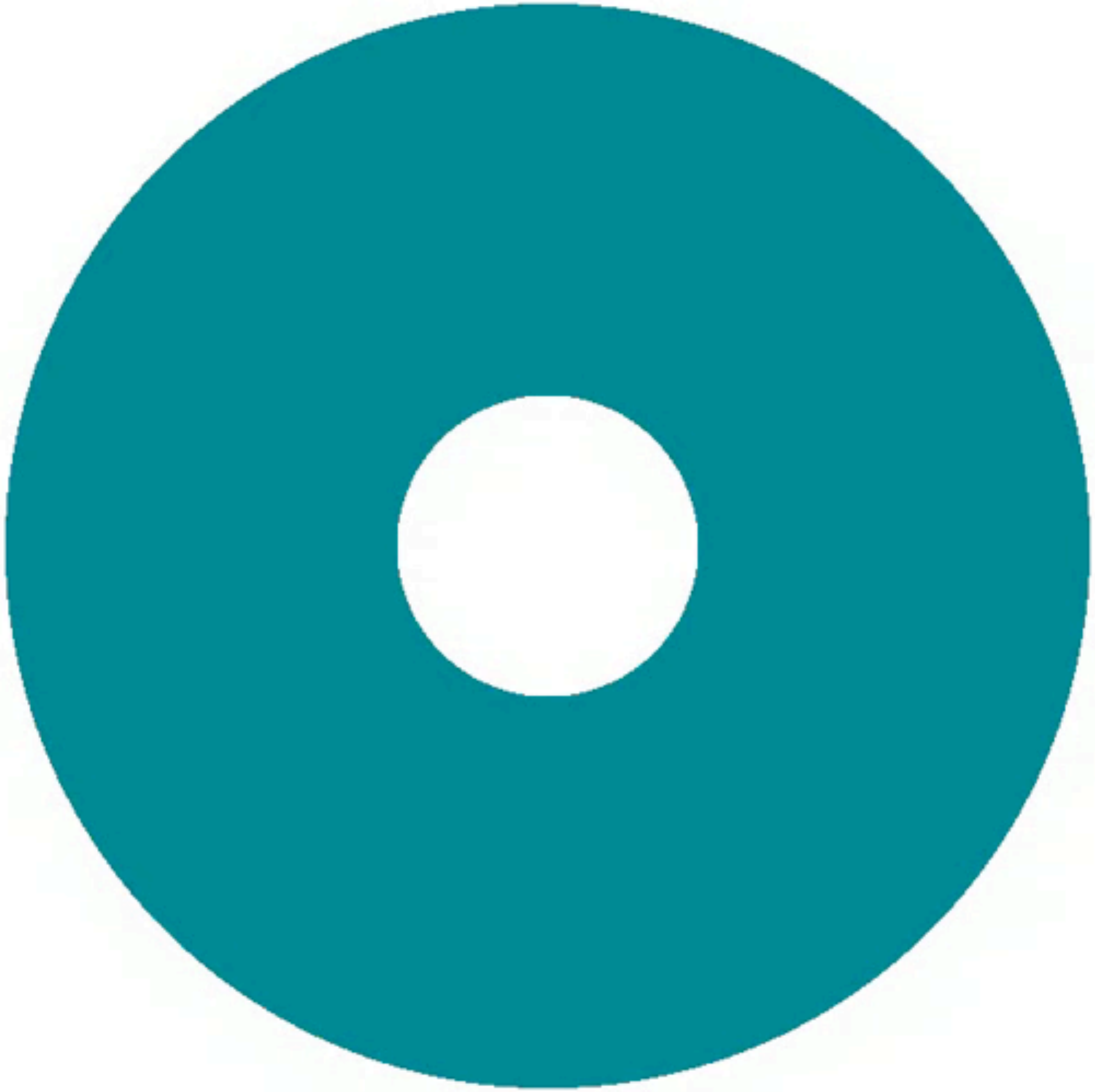
Image credit: NASA/JPL-Caltech

Migration - Type I

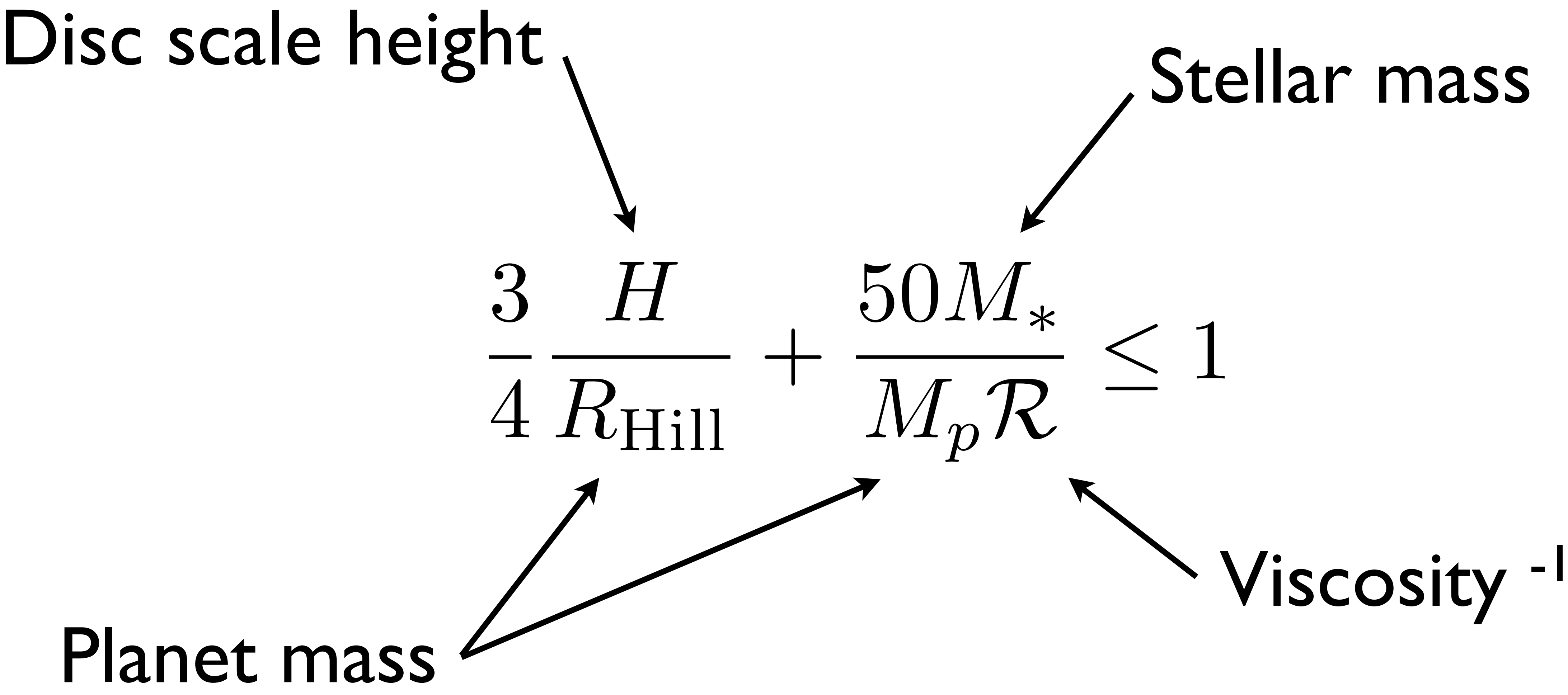
- Low mass planets
- No gap opening in disc
- Migration rate is fast
- Depends strongly on thermodynamics of the disc

Migration - Type II

- Massive planets (typically bigger than Saturn)
- Opens a (clear) gap
- Migration rate is slow
- Follows viscous evolution of the disc



Gap opening criteria



Disc scale height

Stellar mass

$$\frac{3}{4}\frac{H}{R_{\mathrm{Hill}}} + \frac{50 M_*}{M_p \mathcal{R}} \leq 1$$

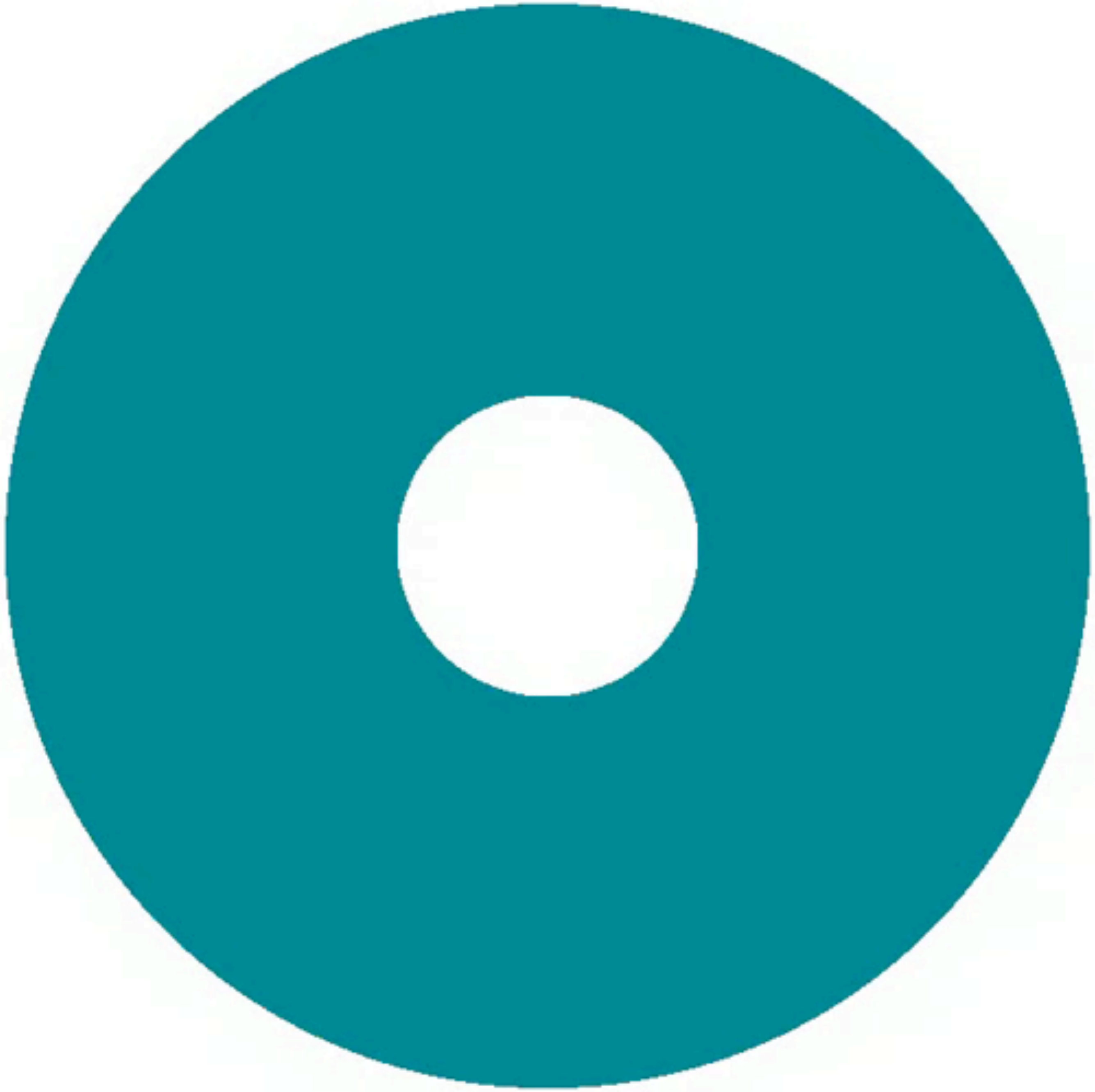
Planet mass

Viscosity $^{-1}$

Crida et al 2006

Migration - Type III

- Massive disc
- Intermediate planet mass
- Tries to open gap
- Very fast, few orbital timescales

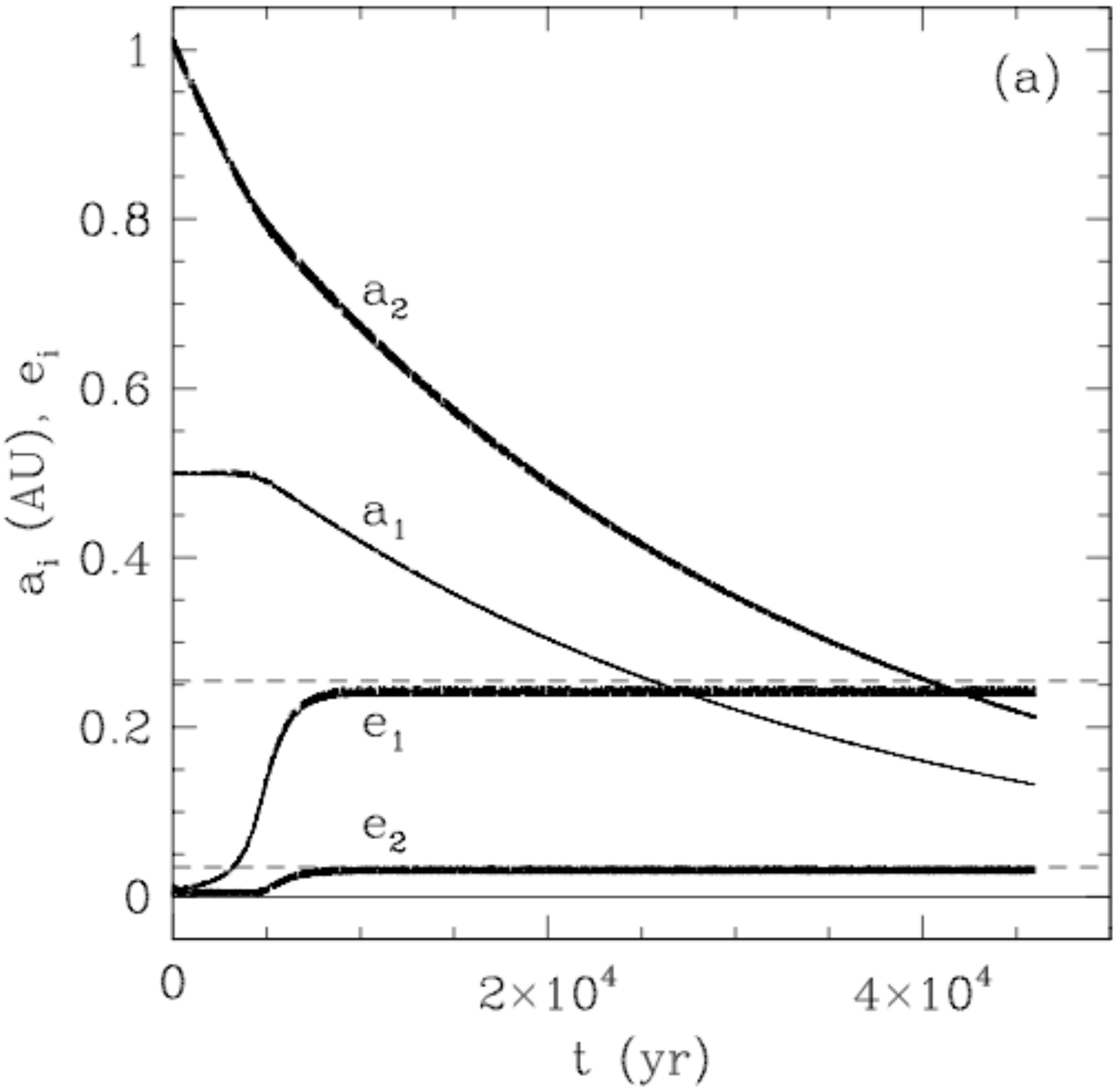


Take home message I

planet + disc = migration

Gliese 876
The role model of resonance capture

GJ 876



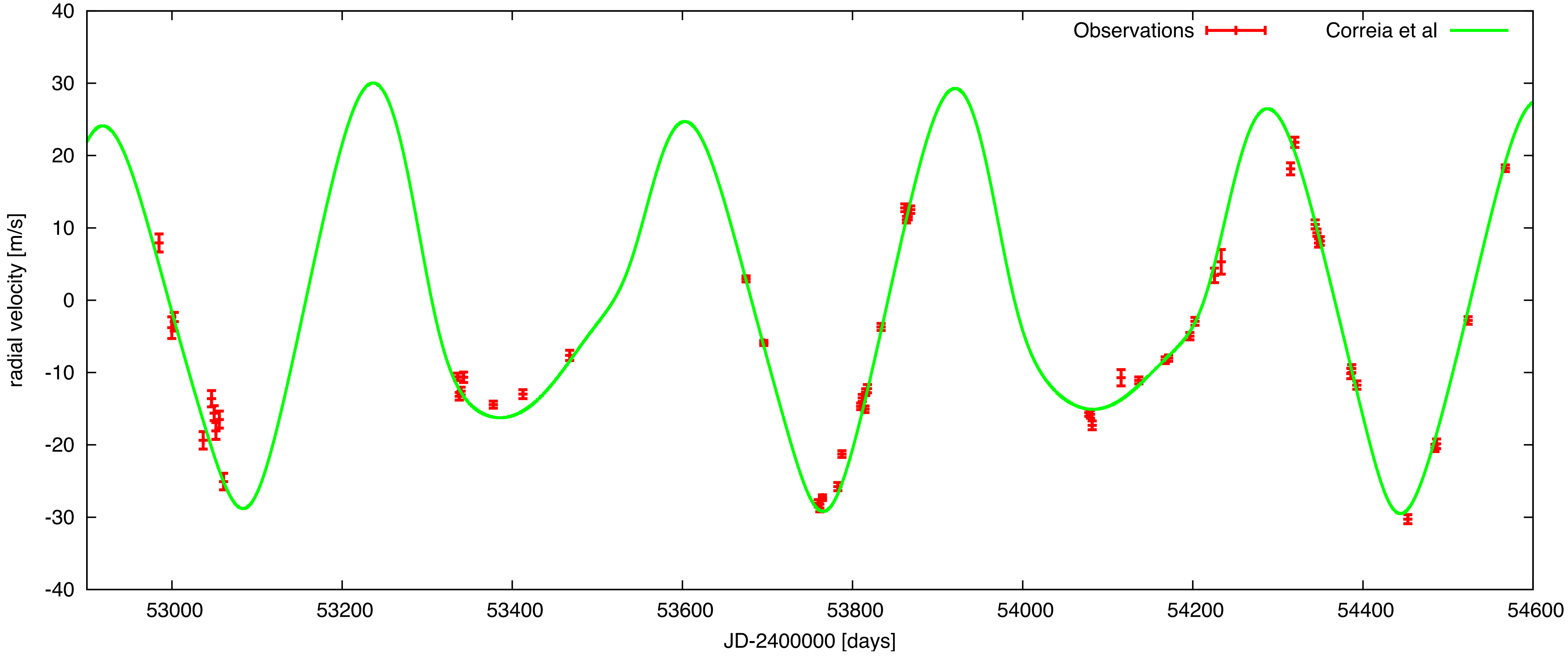
Lee & Peale 2002

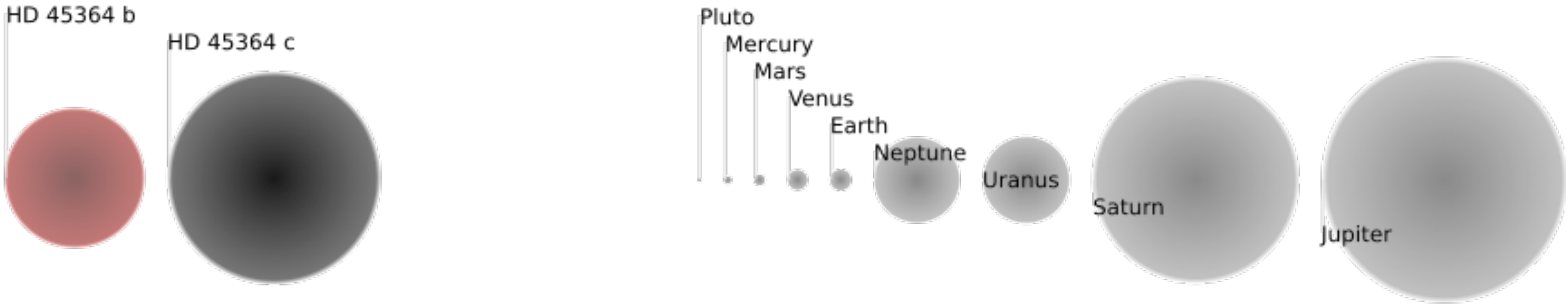
Take home message II

2 planets + migration = resonance

HD 45364
A closely packed system

HD45364





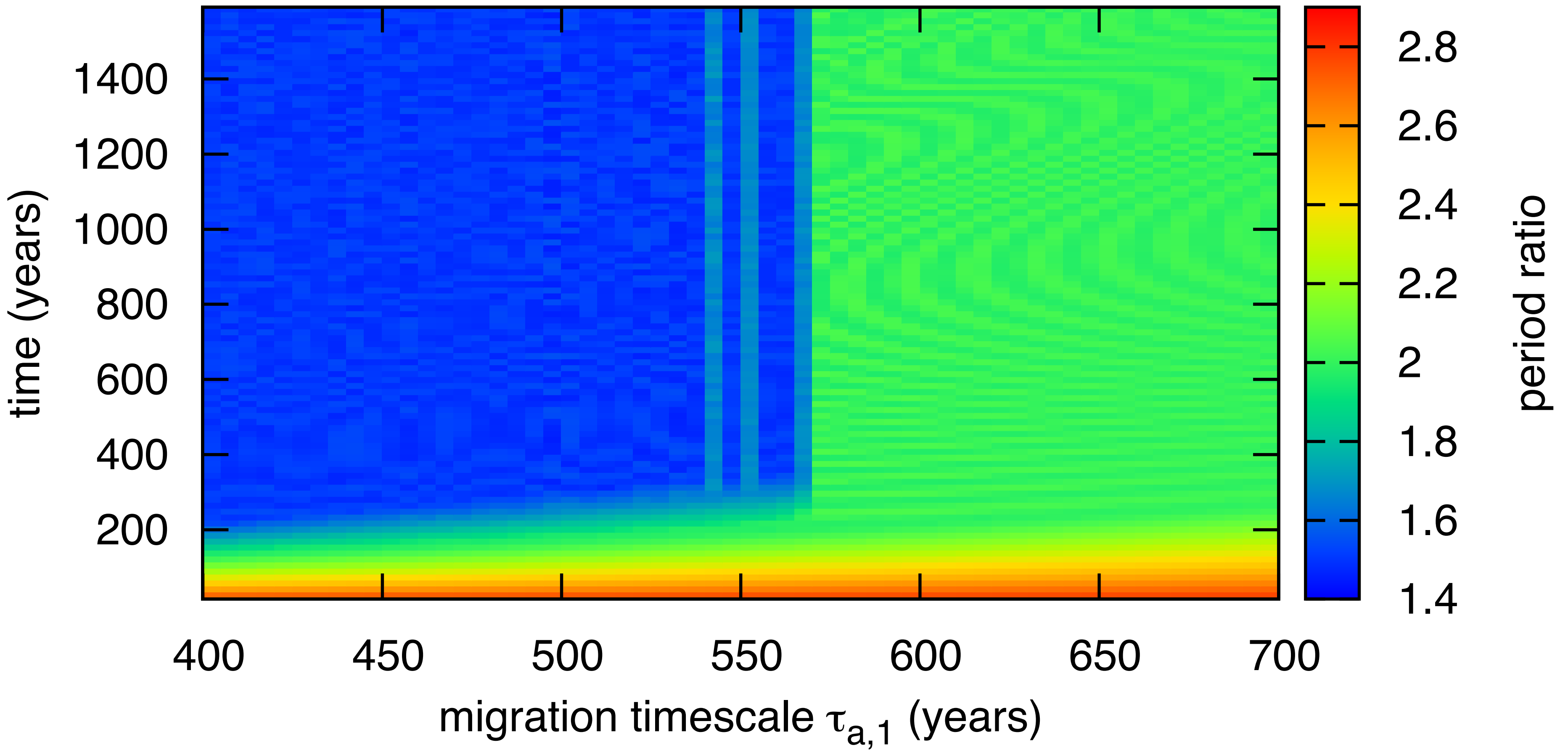
Correia et al 2009, Visual Exoplanet Catalogue

Formation scenario for HD45364

- Two migrating planets
- Infinite number of resonances

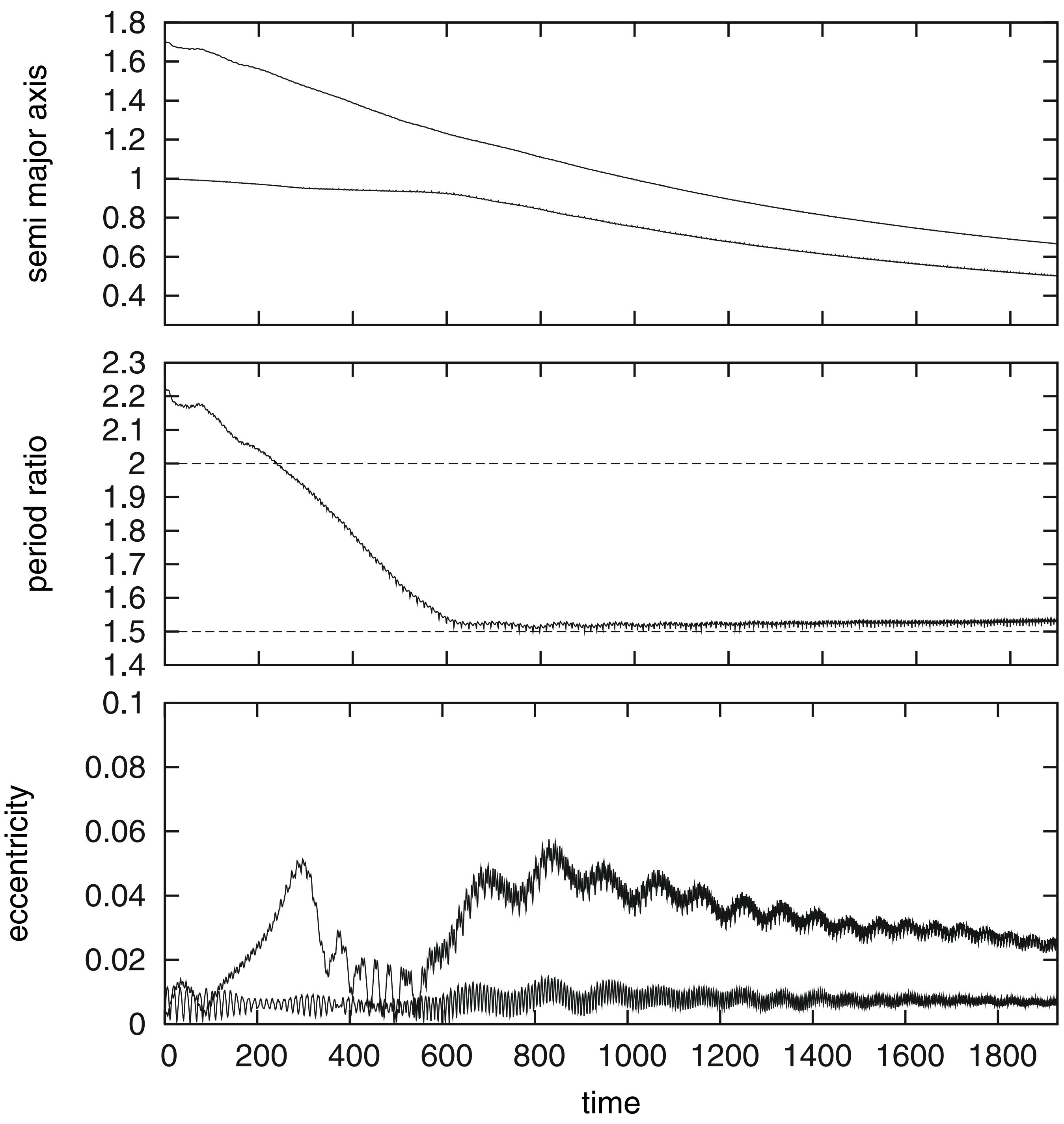
1:2 7:8 3:2 1:3 3:4

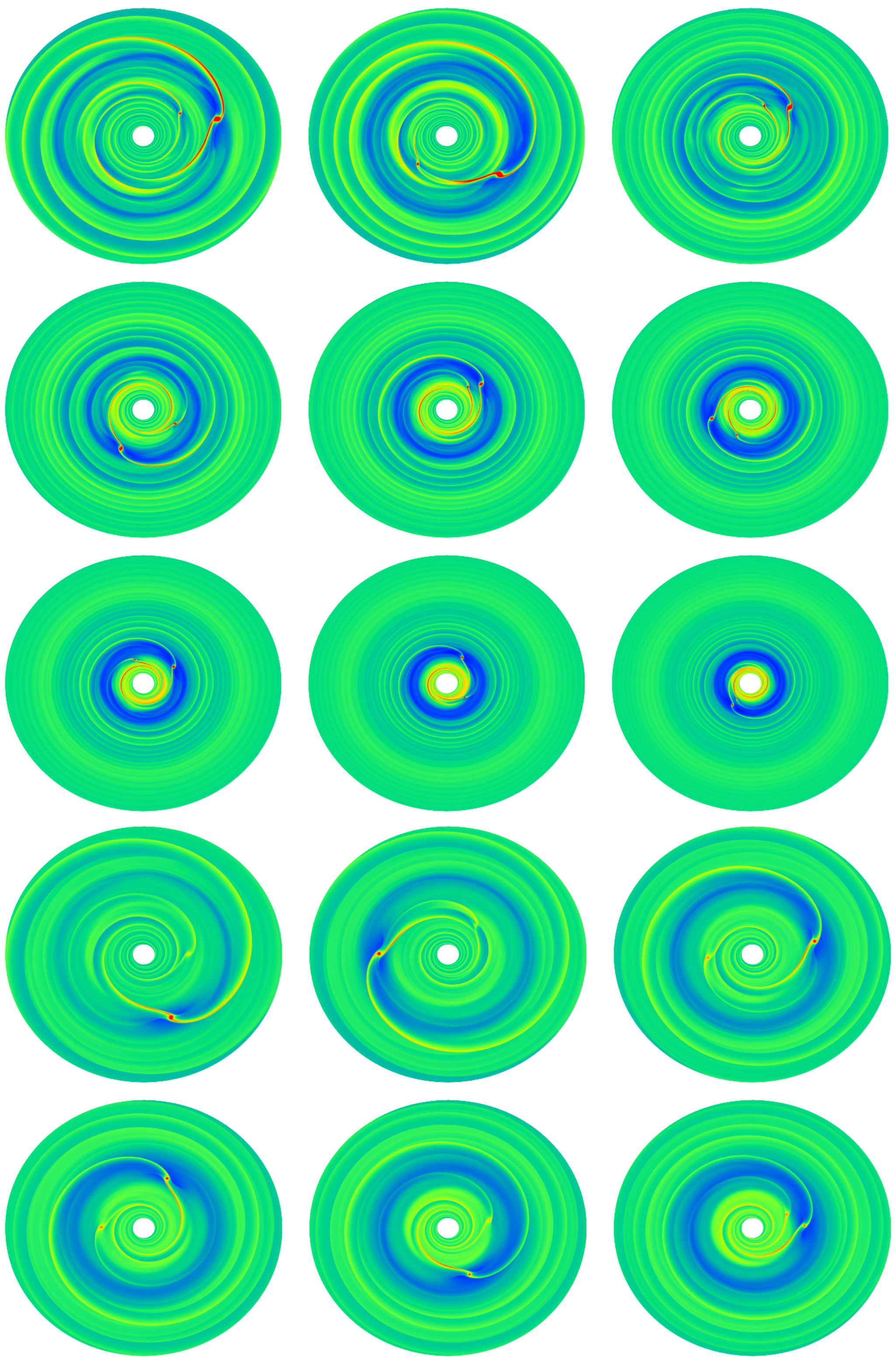
- Migration speed is crucial
- Resonance width and libration period define critical migration rate



Rein, Papaloizou & Kley 2010

Formation scenario for HD45364





Rein, Papaloizou & Kley 2010

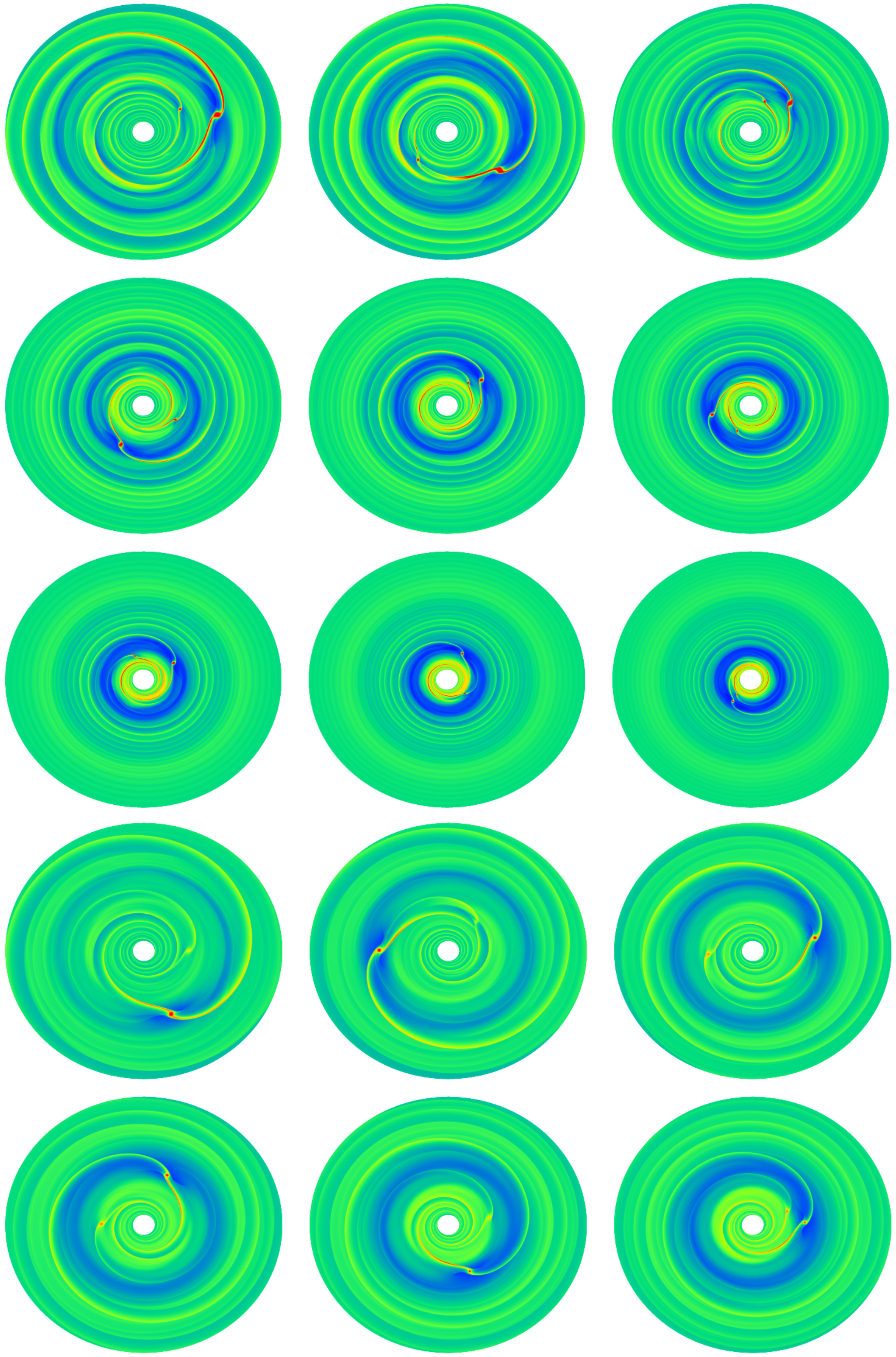
Formation scenario for HD45364

Massive disc (5 times MMSN)

- Short, rapid Type III migration
- Passage of 2:1 resonance
- Capture into 3:2 resonance

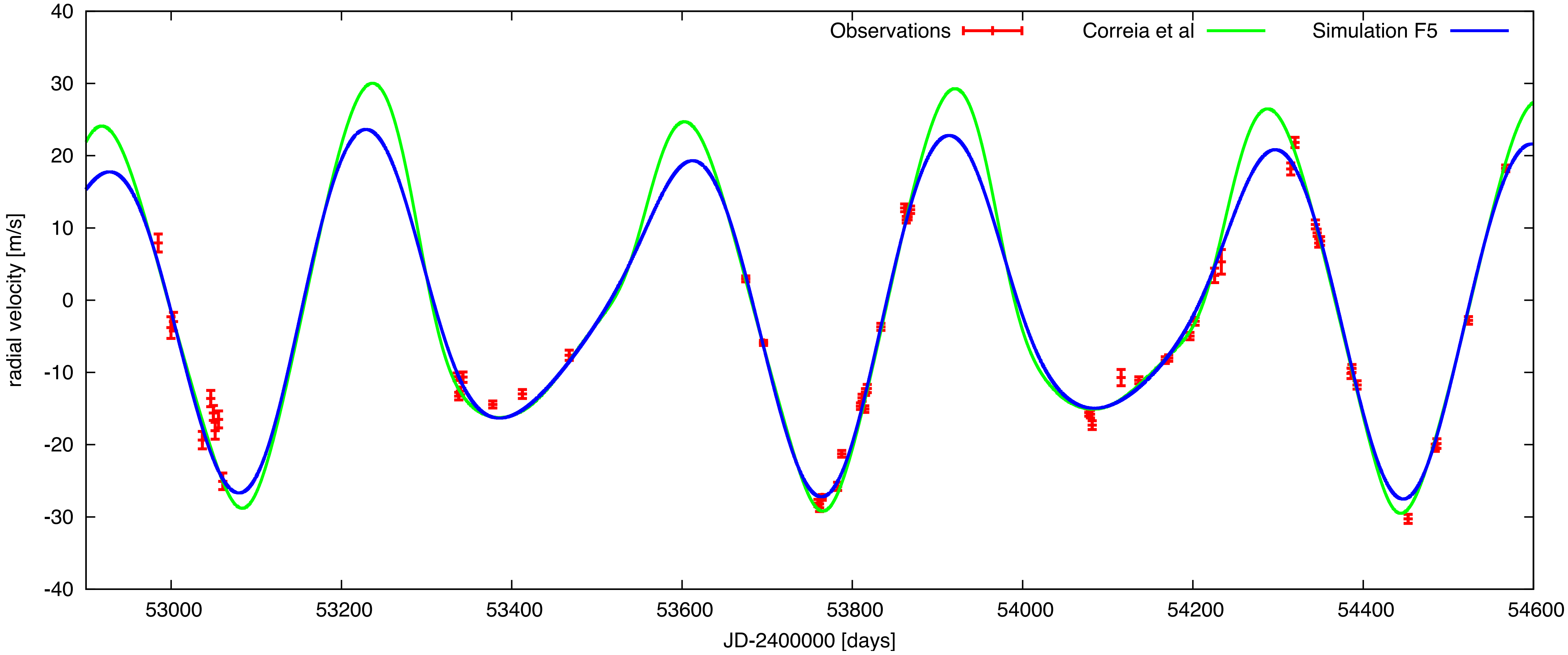
Large scale-height (0.07)

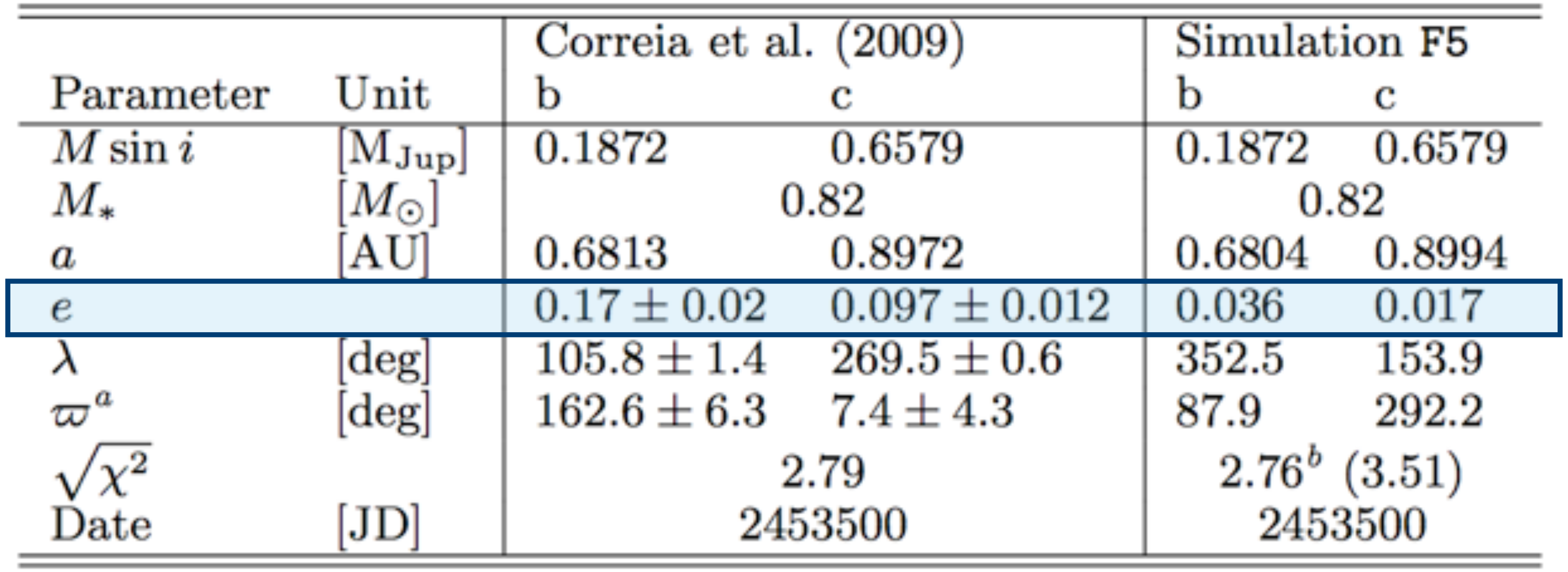
- Slow Type I migration once in resonance
- Resonance is stable
- Consistent with radiation hydrodynamics



Rein, Papaloizou & Kley 2010

Formation scenario leads to a better ‘fit’





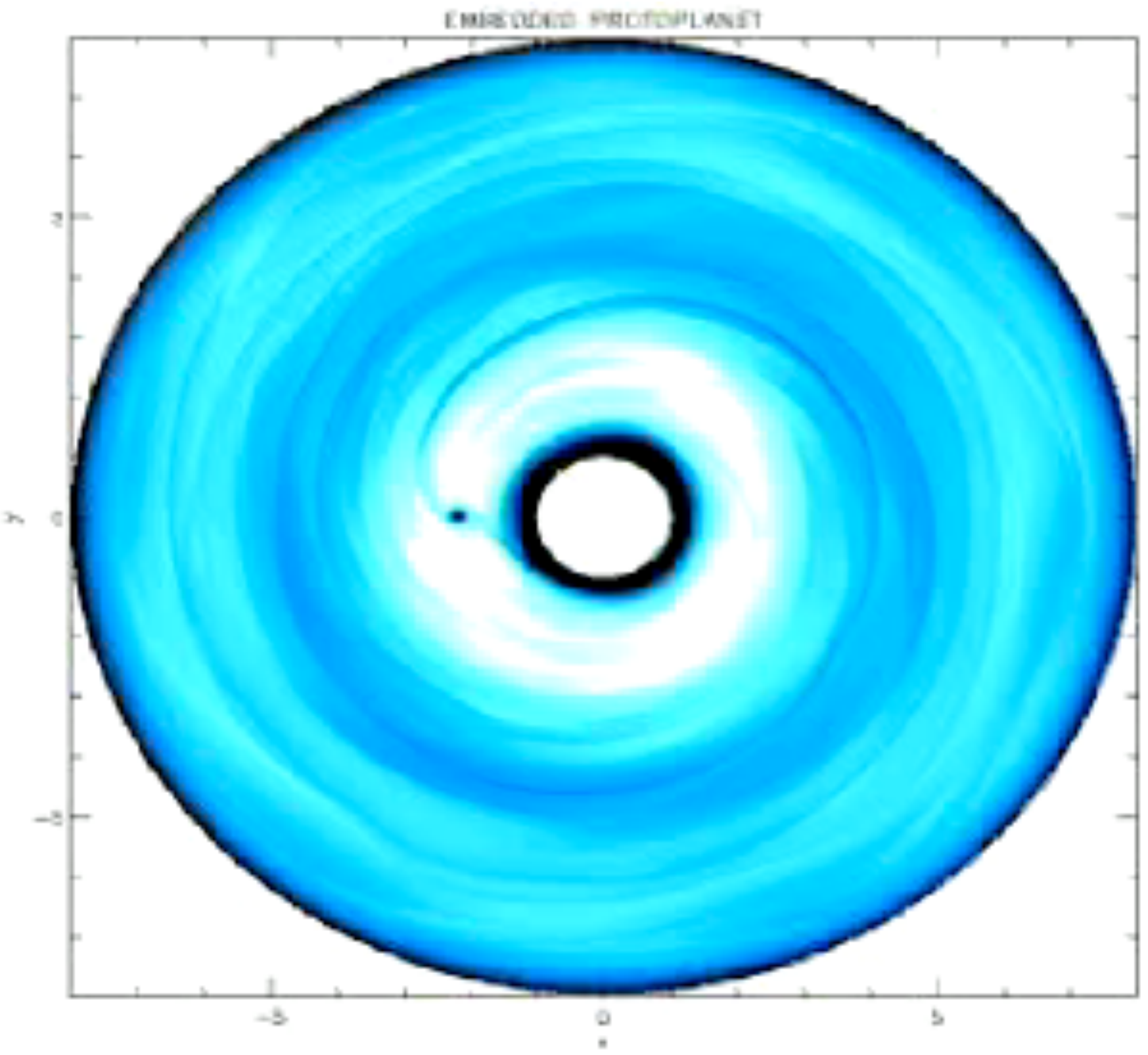
Parameter	Unit	Correia et al. (2009)		Simulation F5	
		b	c	b	c
$M \sin i$	$[\mathrm{M_{Jup}}]$	0.1872	0.6579	0.1872	0.6579
M_*	$[M_\odot]$	0.82		0.82	
a	[AU]	0.6813	0.8972	0.6804	0.8994
e		0.17 ± 0.02	0.097 ± 0.012	0.036	0.017
λ	[deg]	105.8 ± 1.4	269.5 ± 0.6	352.5	153.9
ϖ^a	[deg]	162.6 ± 6.3	7.4 ± 4.3	87.9	292.2
$\sqrt{\chi^2}$		2.79		2.76^b (3.51)	
Date	[JD]	2453500		2453500	

Rein, Papaloizou & Kley 2010

HD 128311
Migration in a turbulent disc

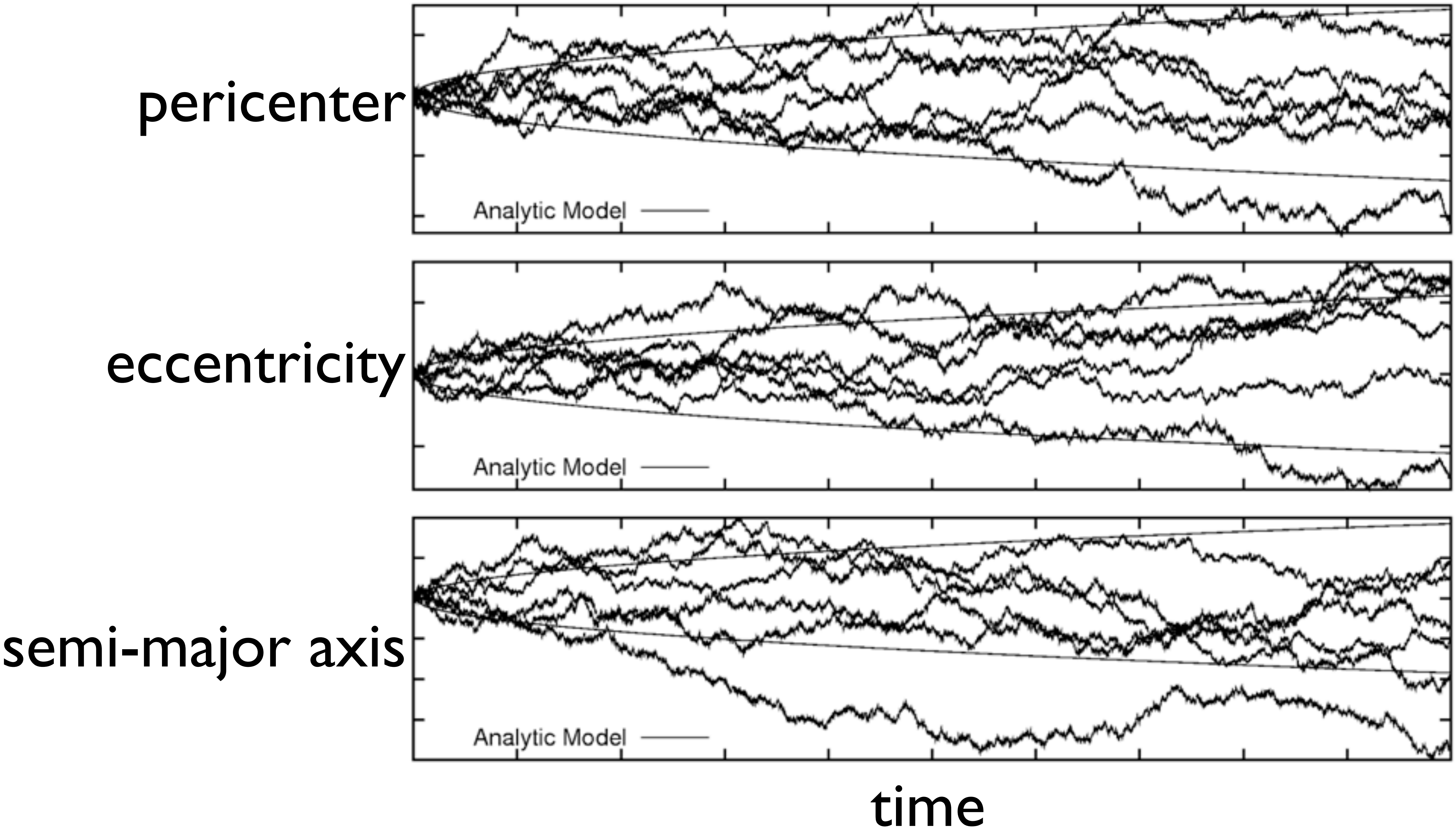
Turbulent disc

- Angular momentum transport
- Magnetorotational instability (MRI)
- Density perturbations interact gravitationally with planets
- Stochastic forces lead to random walk
- Large uncertainties in strength of forces



Animation from Nelson & Papaloizou 2004
Random forces measured by Laughlin et al. 2004, Nelson 2005, Oischi et al. 2007

Random walk



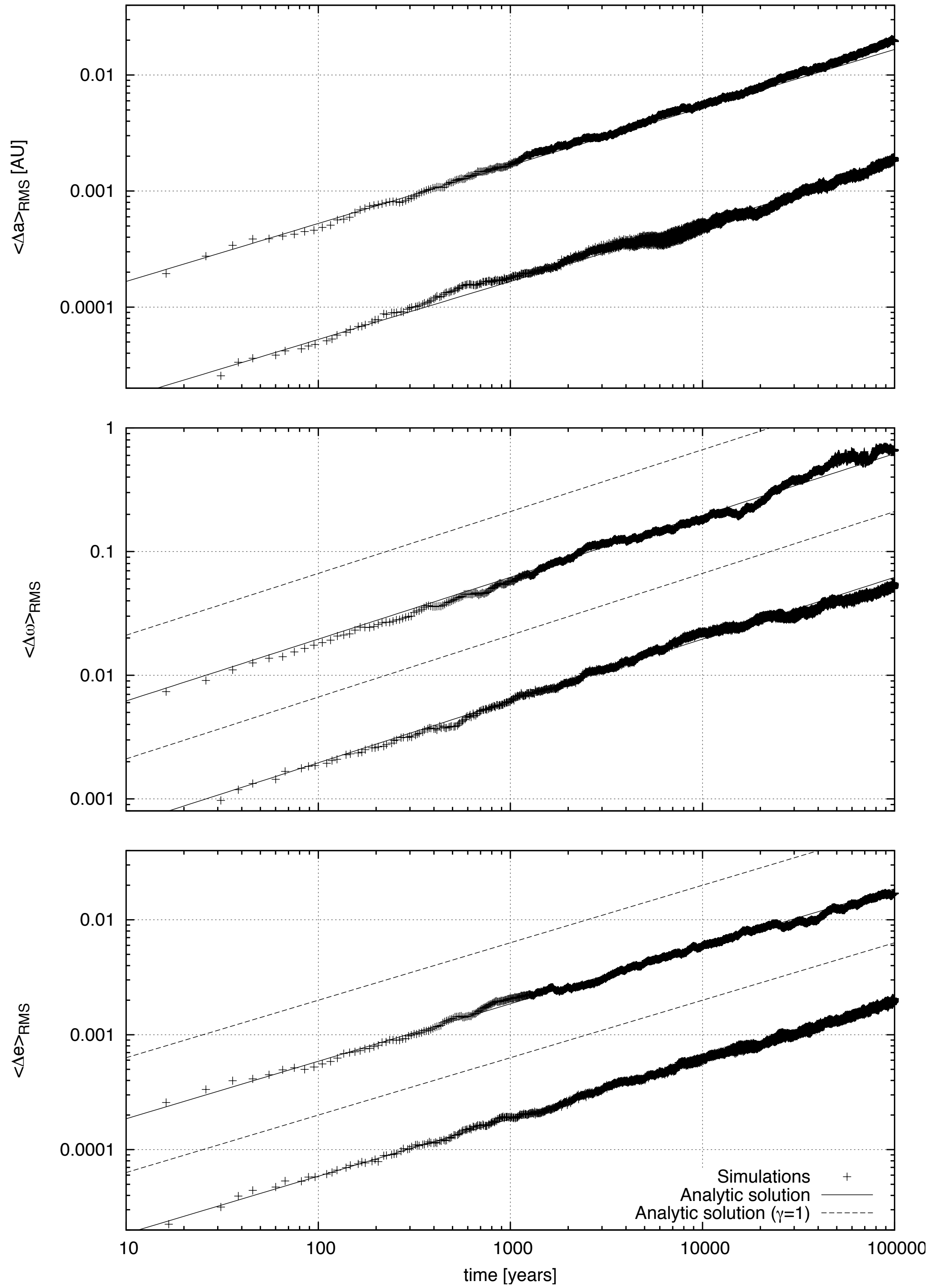
Rein & Papaloizou 2009

Correction factors are important

$$(\Delta a)^2 = 4\frac{Dt}{n^2}$$

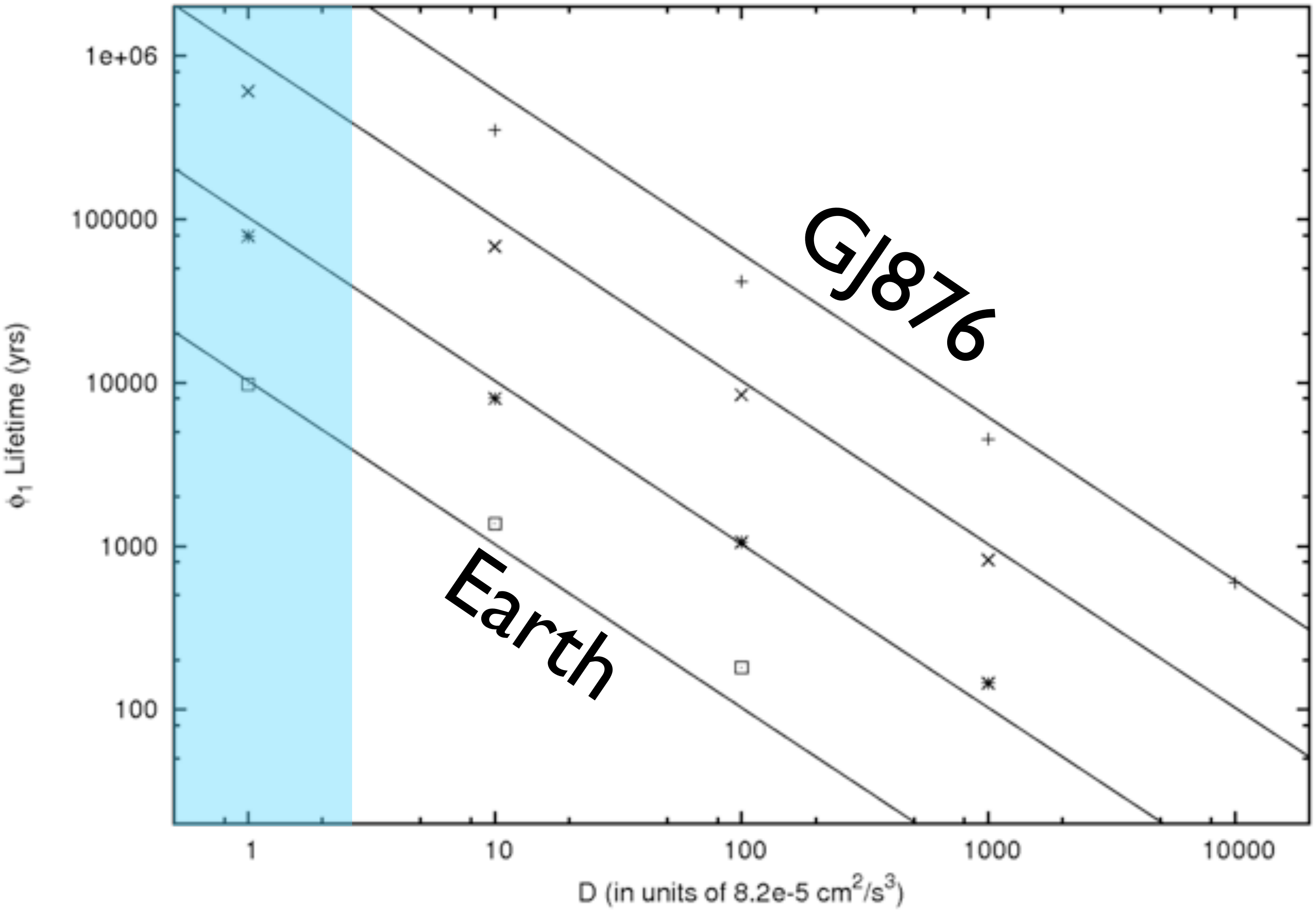
$$(\Delta \varpi)^2 = \frac{2.5}{e^2}\,\frac{\gamma D t}{n^2 a^2}$$

$$(\Delta e)^2 = 2.5\frac{\gamma D t}{n^2 a^2}$$



Rein & Papaloizou 2009, Adams et al 2009, Rein 2010

Multi-planetary systems in mean motion resonance

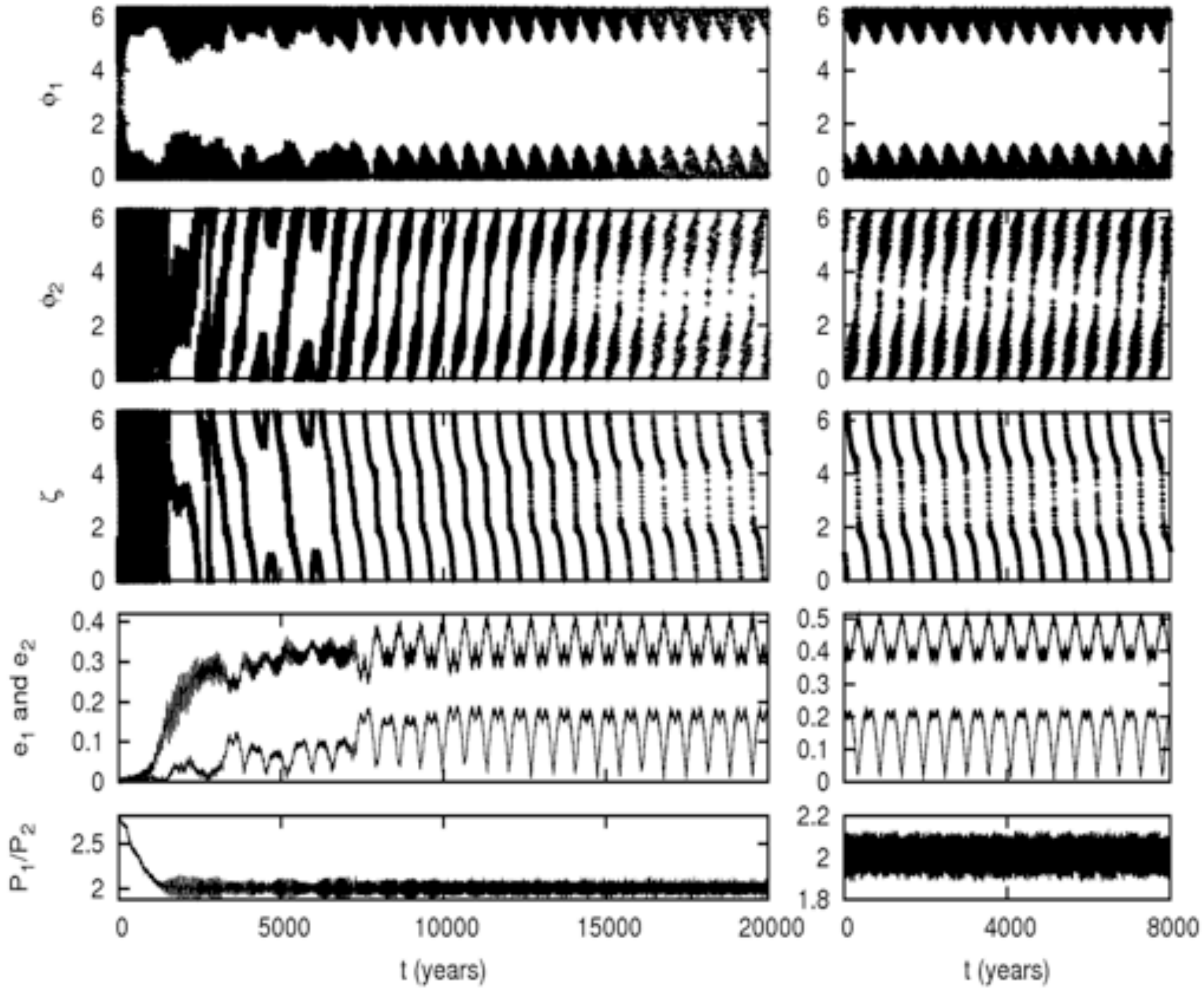


- Stability of multi-planetary systems depends strongly on diffusion coefficient
- Most planetary systems are stable for entire disc lifetime

Rein & Papaloizou 2009

Modification of libration patterns

- HD128311 has a very peculiar libration pattern
- Can not be reproduced by convergent migration alone
- Turbulence can explain it
- More multi-planetary systems needed for statistical argument



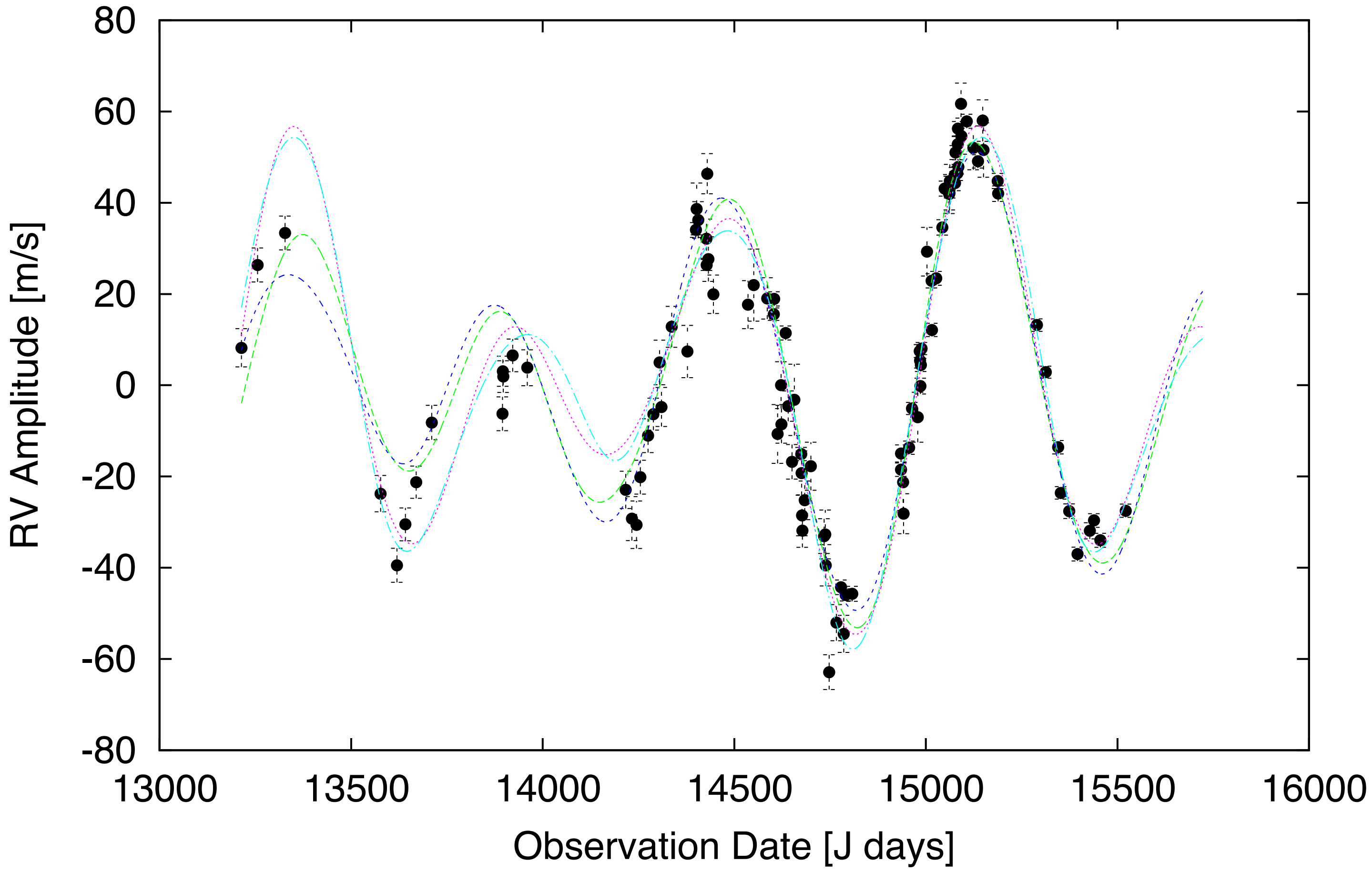
Rein & Papaloizou 2009

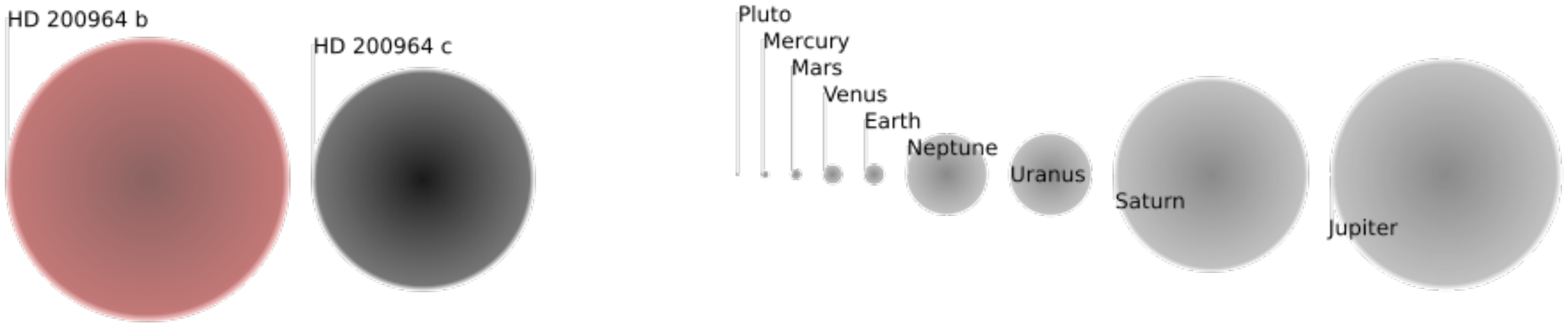
Take home message III

Migration scenarios can explain the dynamical configuration of many systems in amazing detail

HD200964
The impossible system

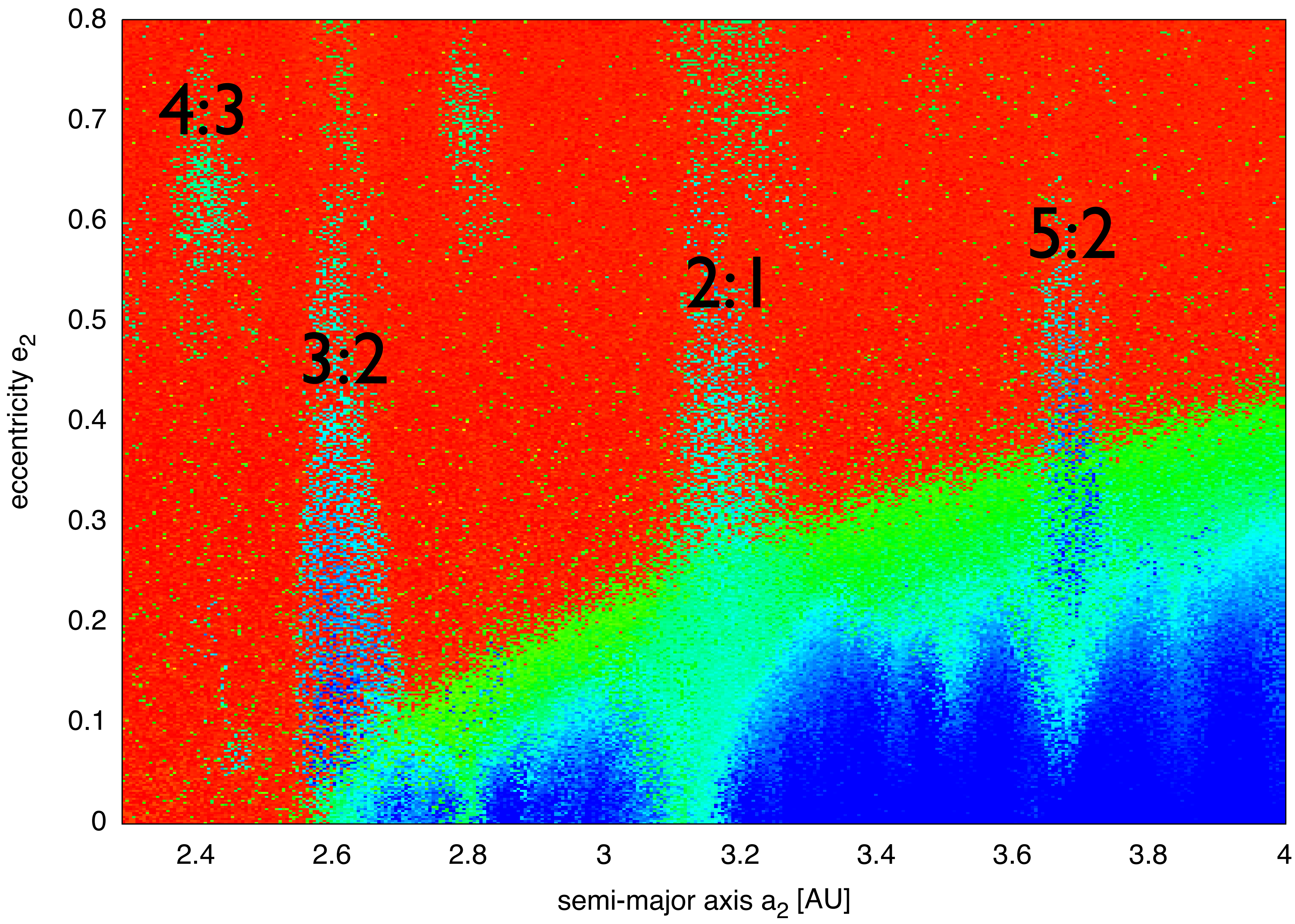
Radial velocity curve of HD200964





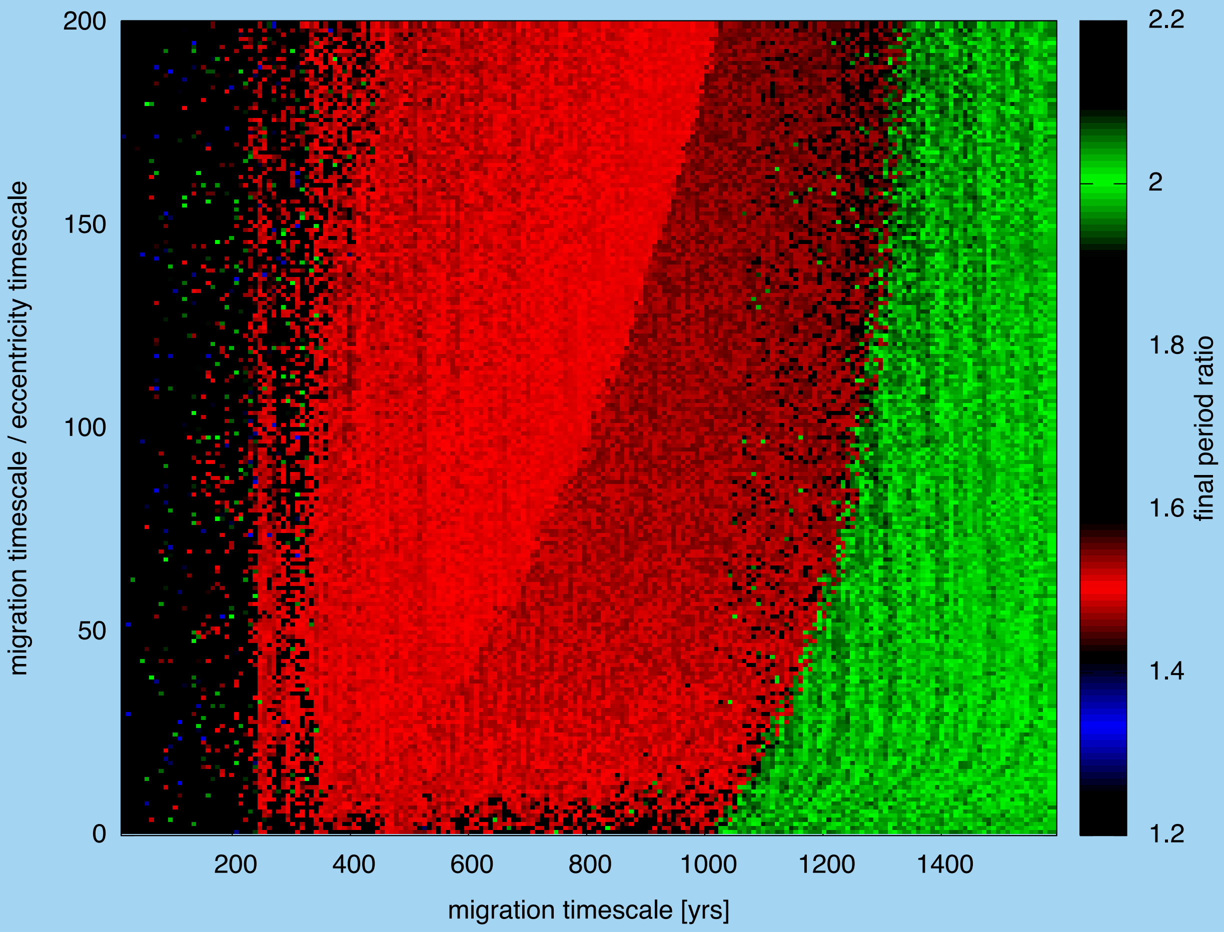
Plot by Matthew Payne

Stability of HD200964

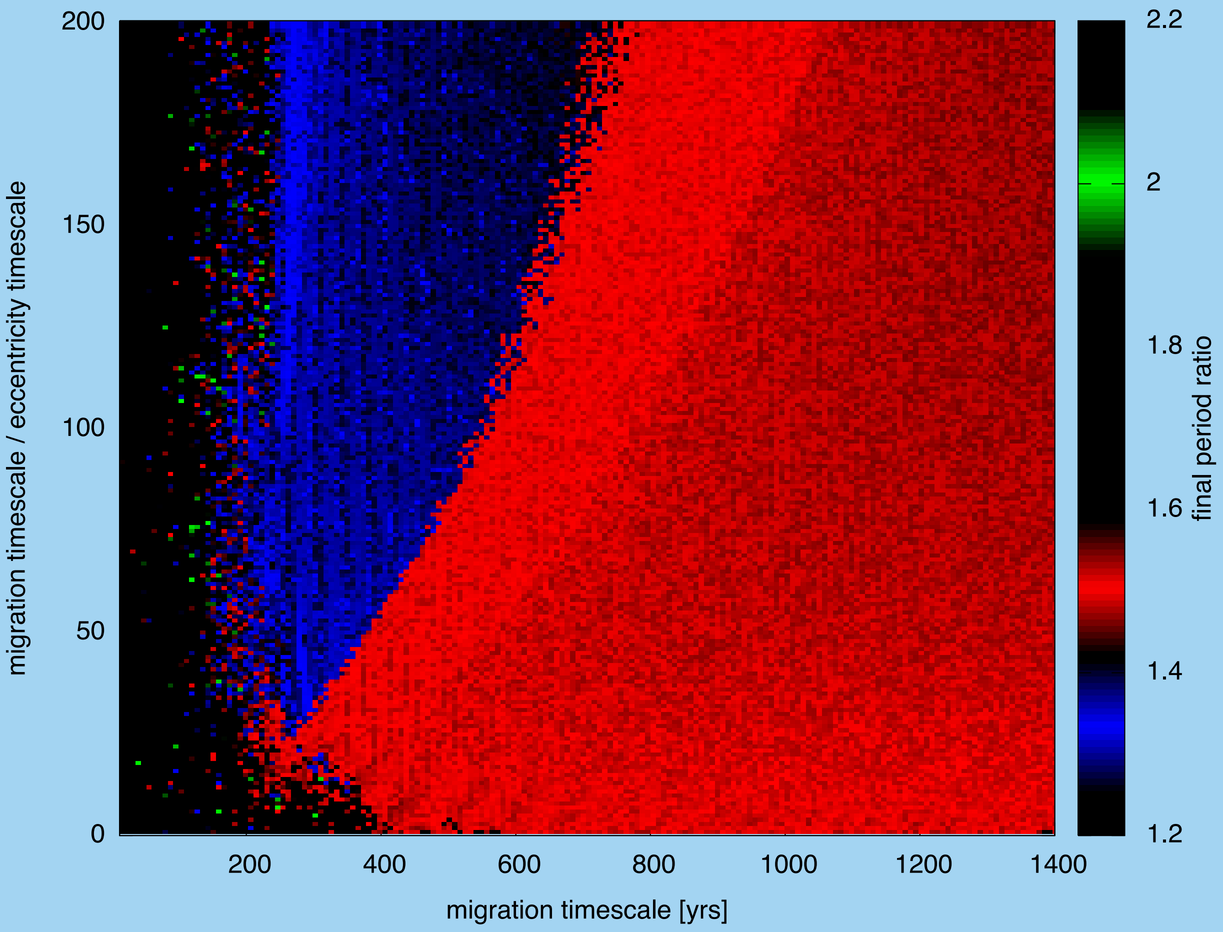


Standard disc migration

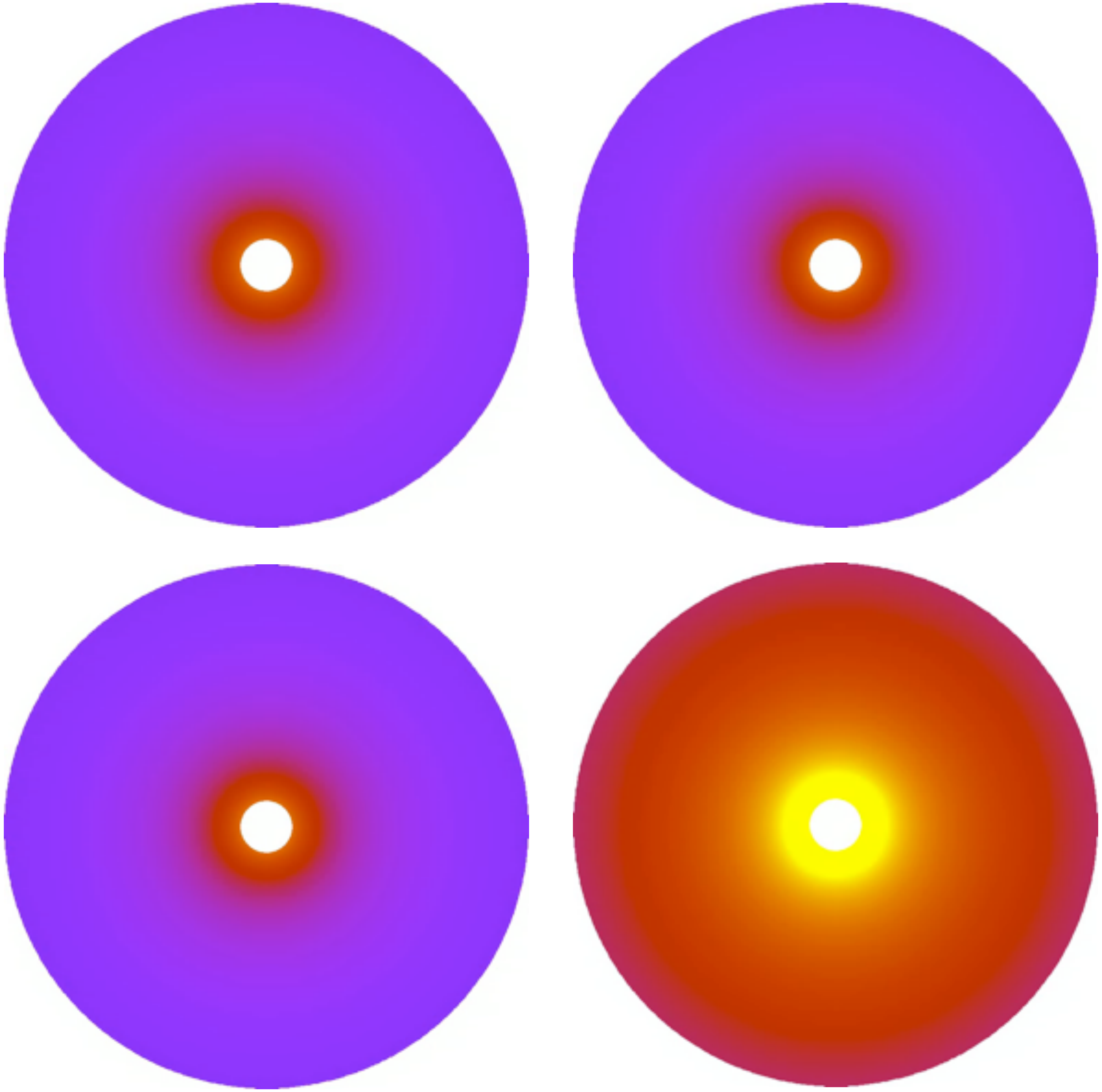
Observed (minimum) masses



Reduced masses



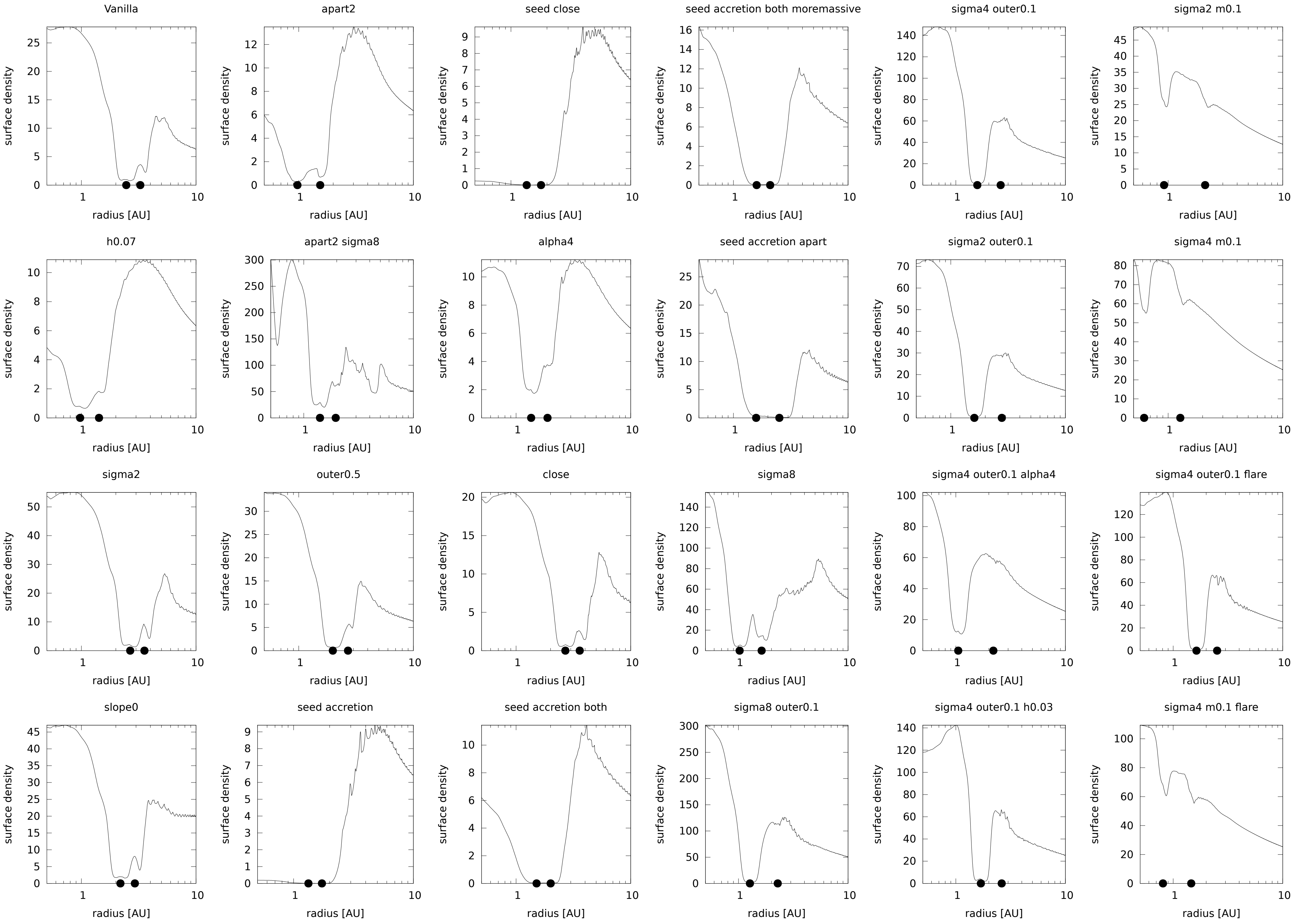
Standard disc migration



In addition to N-body simulations, we ran almost 100 hydrodynamic simulations

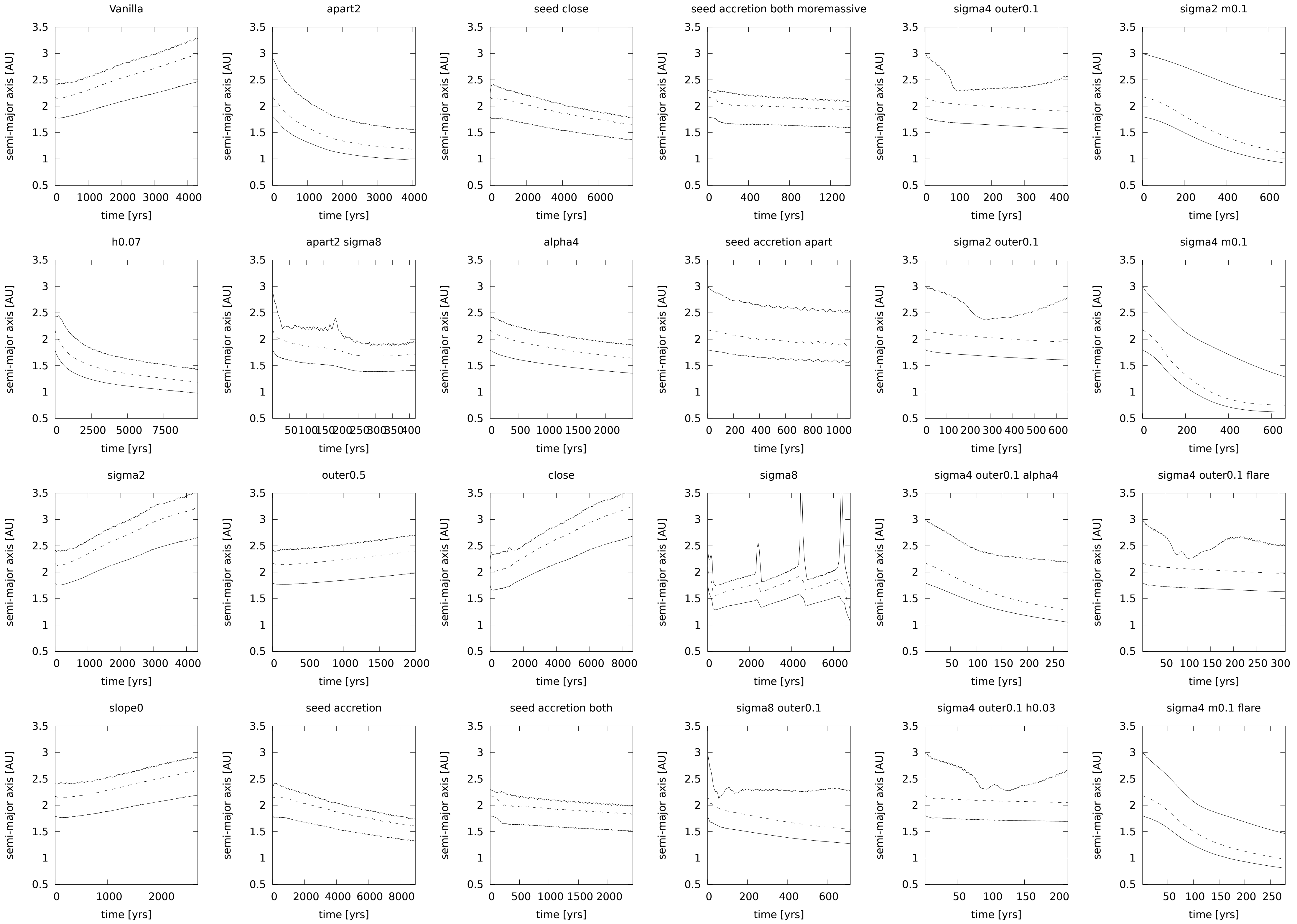
Experiments with many different parameters: surface density, slope, scale height, viscosity, planet masses, boundaries, accretion, ...

Hydrodynamical simulations II



Rein, Payne, Vera & Ford (2012 in prep)

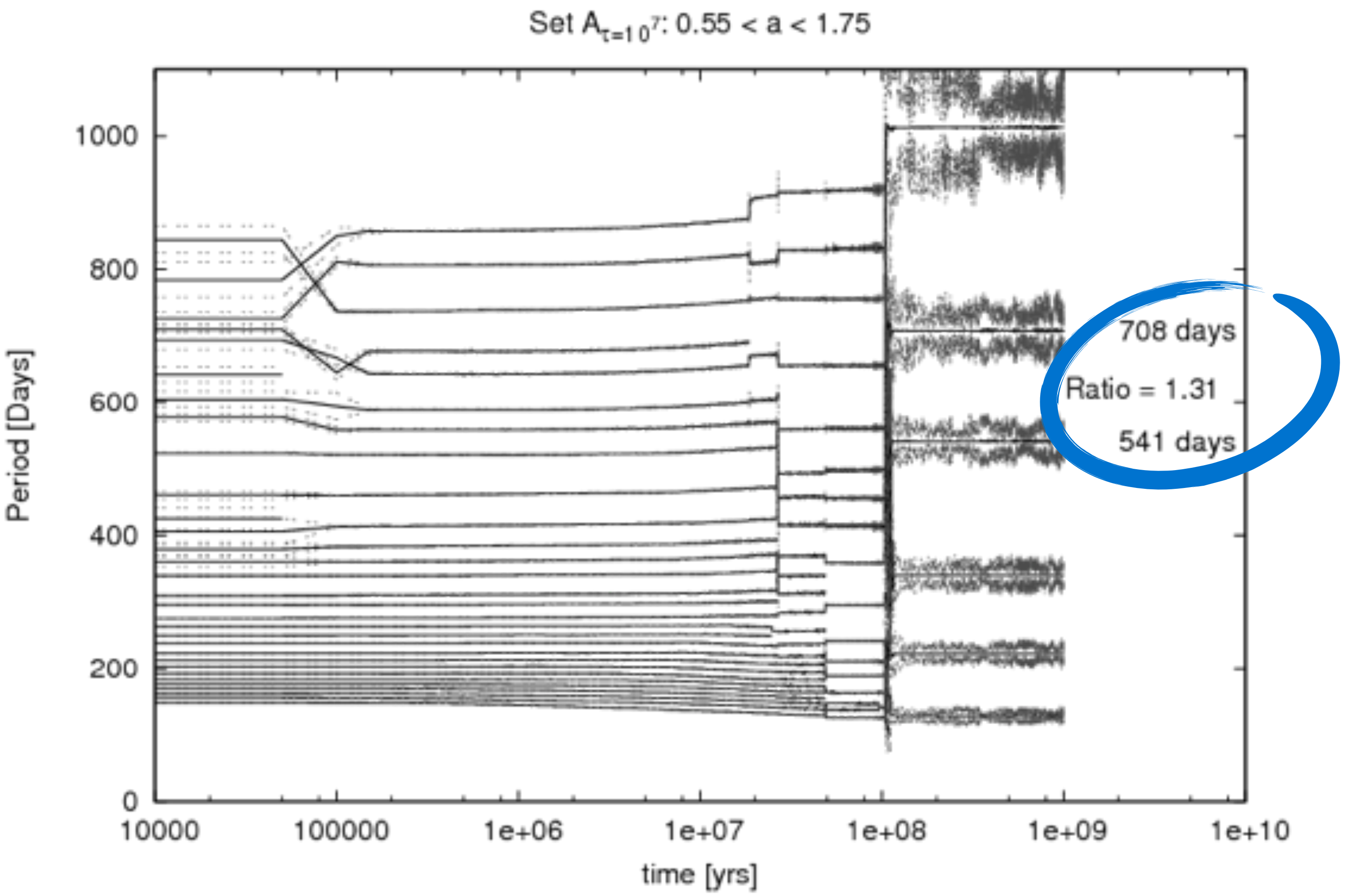
Hydrodynamical simulations III



Vanilla
apart2
seed close
seed accretion both moremassive
sigma4 outer0.1
sigma2 m0.1
h0.07
apart2 sigma8
alpha4
seed accretion apart
sigma2 outer0.1
sigma4 m0.1
sigma2
outer0.5
close
sigma8
sigma4 outer0.1 alpha4
sigma4 outer0.1 flare
slope0
seed accretion
seed accretion both
sigma8 outer0.1
sigma4 outer0.1 h0.03
sigma4 m0.1 flare

Rein, Payne, Vera & Ford (2012 in prep)

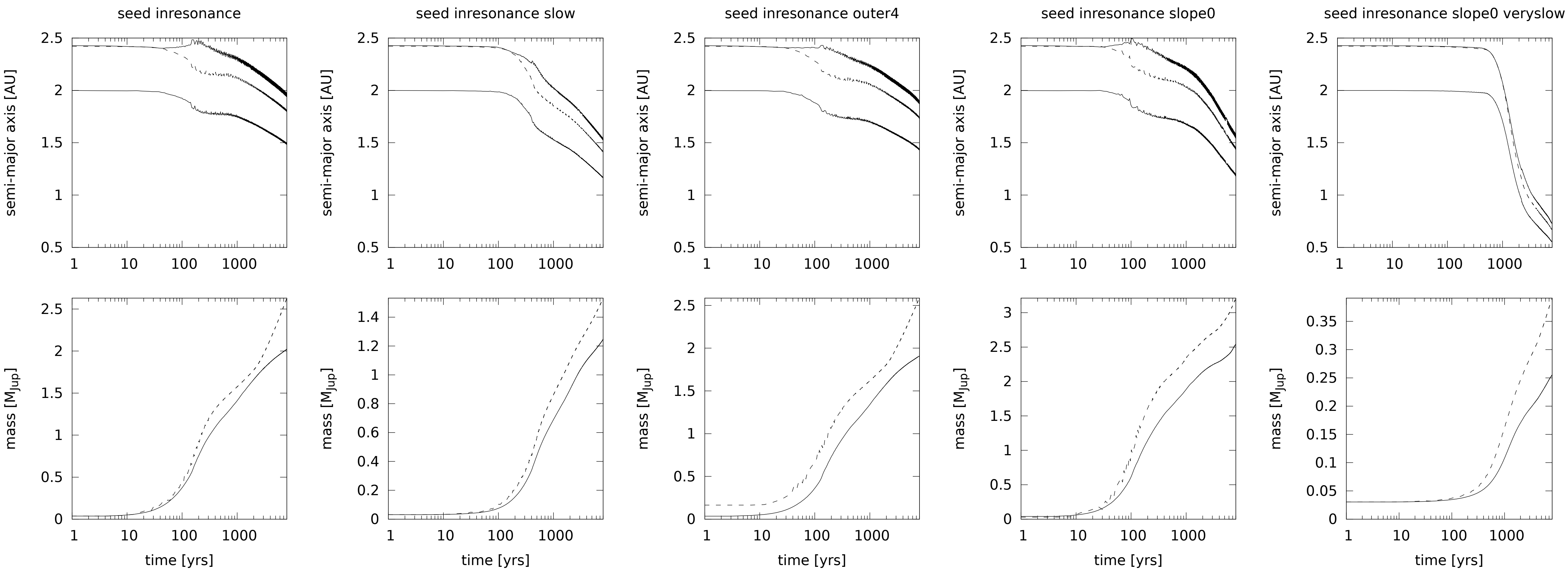
Scattering of embryos



Finite number of embryos end up in close in resonances during oligarchic growth phase.

Rein, Payne, Vera & Ford (2012 in prep)

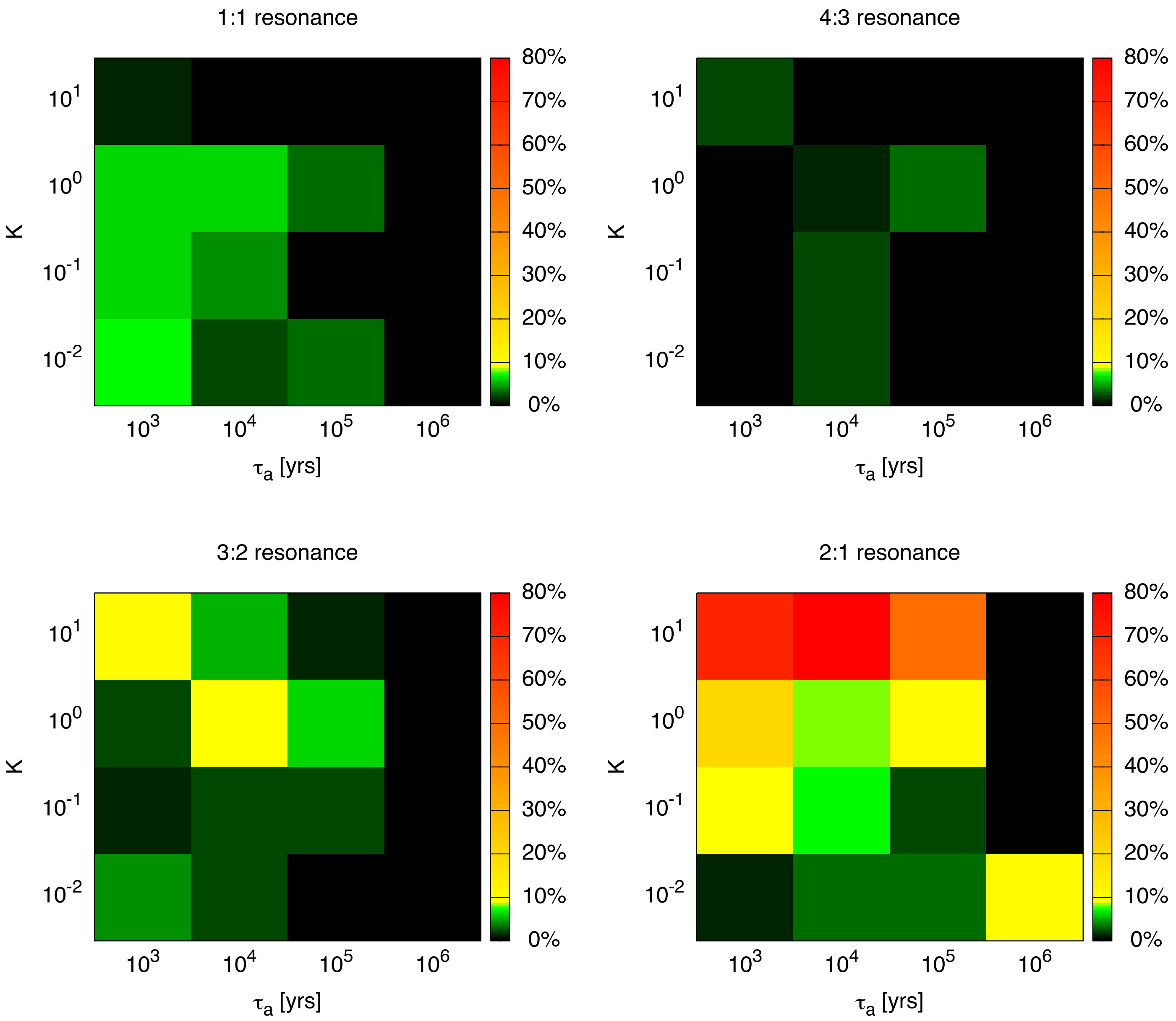
Embryos in a gas disk



Initially in resonance

Resonance lost quickly because of migration and accretion

Scattering and damping



Fine tuned planet-planet scattering simulations

Only a very small fraction ends up in 4:3 resonance

Many more end up in 1:1 resonances, inconsistent with observations

HD200964

Migration

Wrong resonance

In-situ formation

Scattering

Planet-planet scattering

Probability too low

RV signal due to additional planets

Inconsistent

Observers screwed up

N > 1

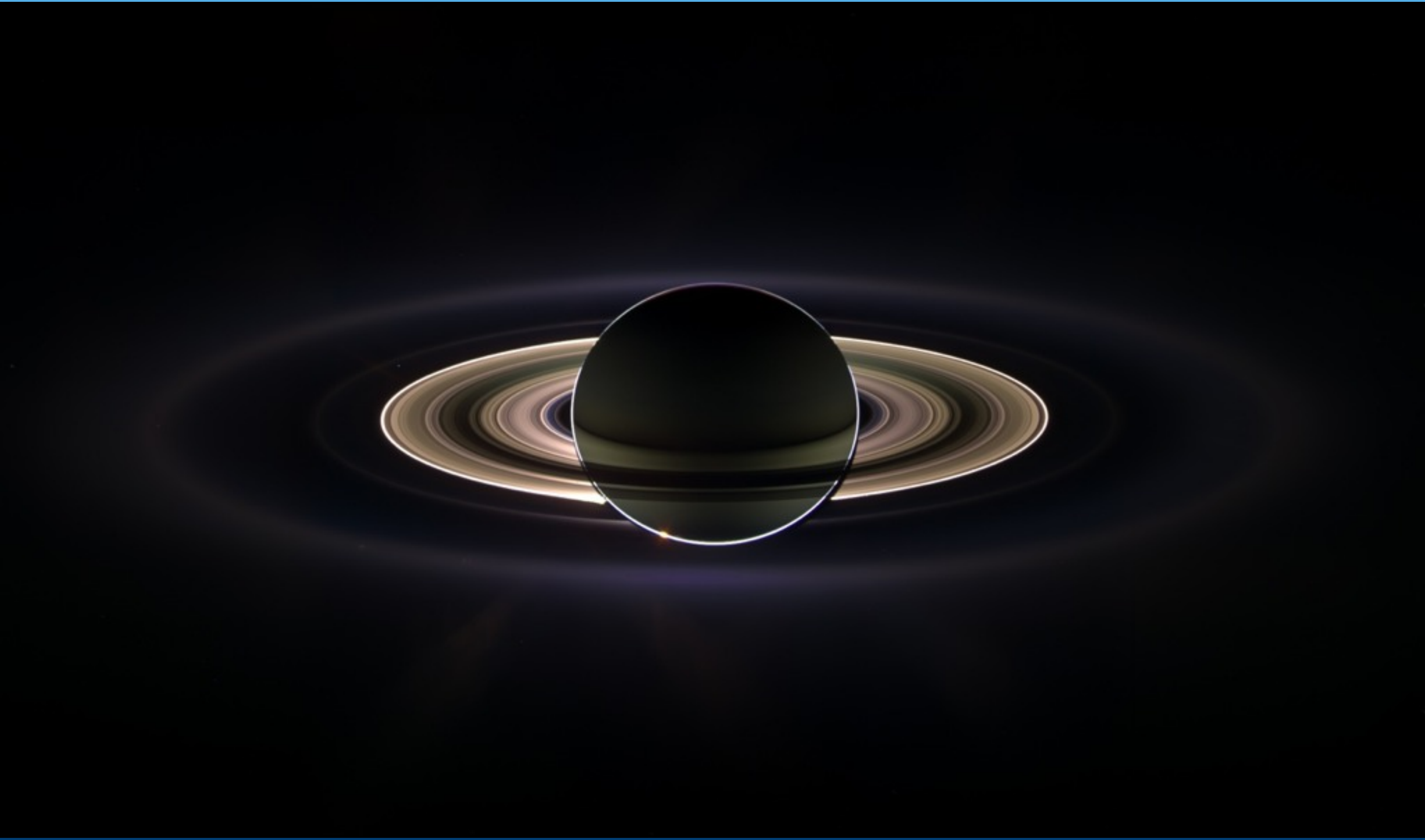
?

Take home message IV

There is still a lot that we do not understand

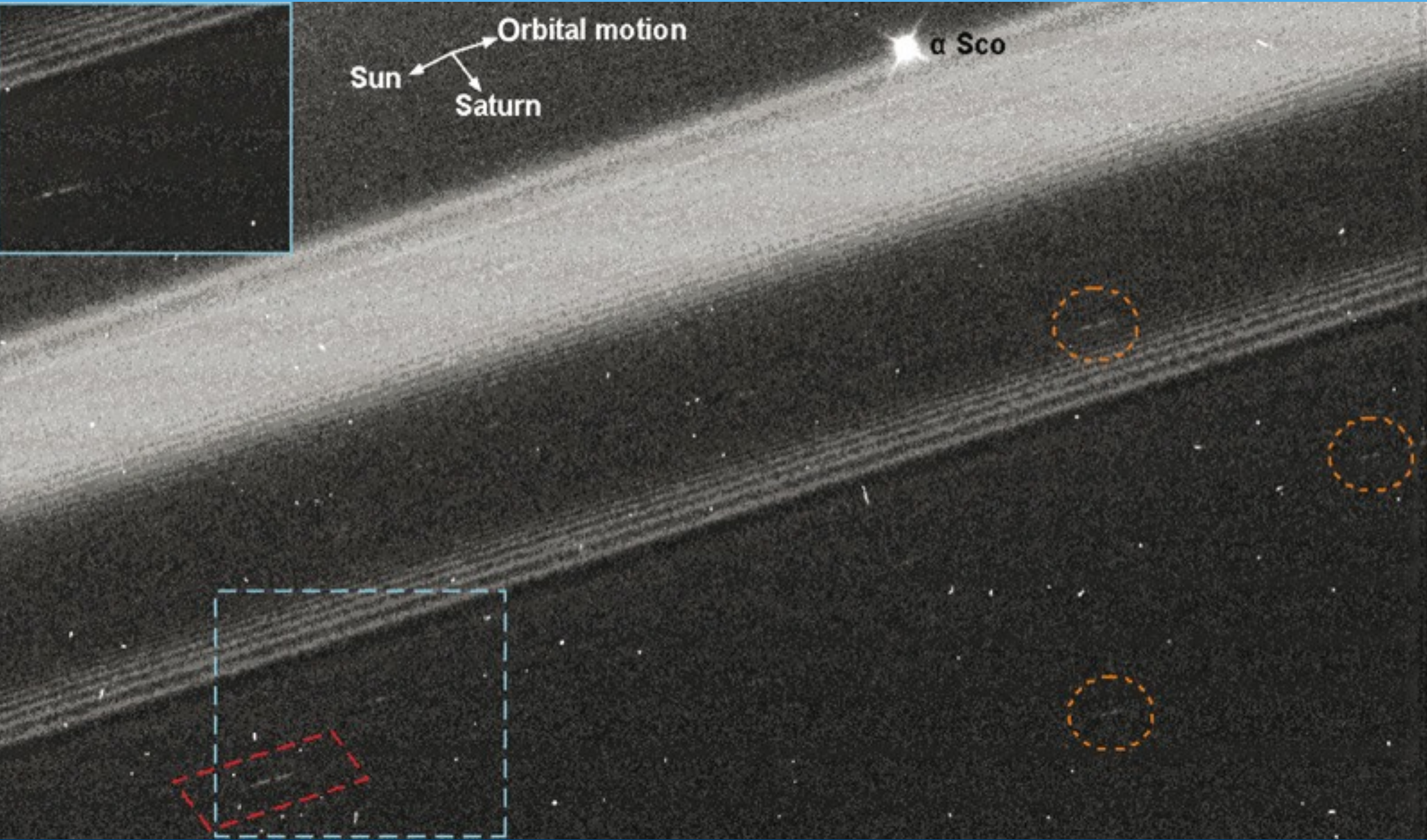
Moonlets in Saturn's Rings

Cassini spacecraft



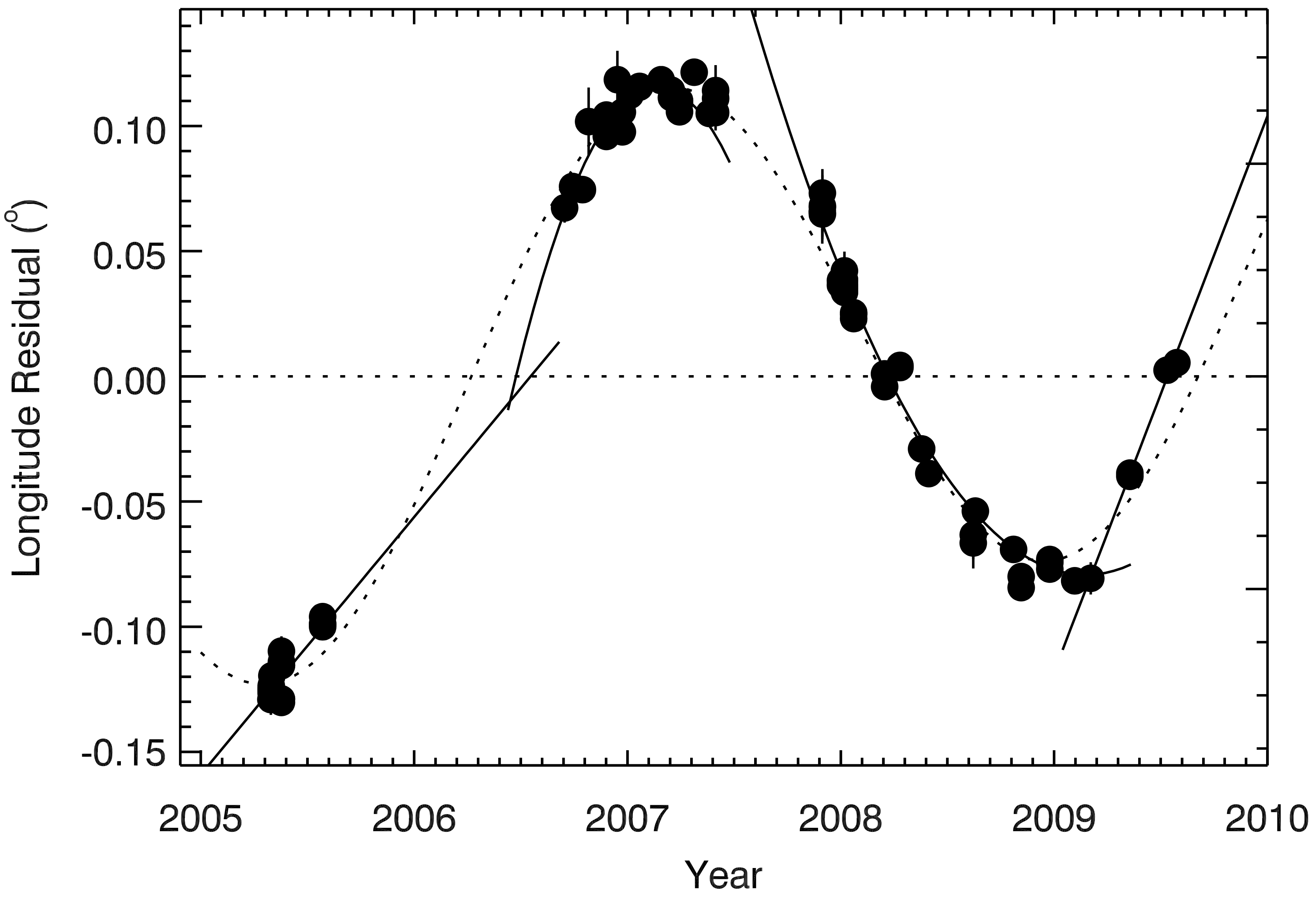
NASA/JPL/Space Science Institute

Propeller structures in A-ring



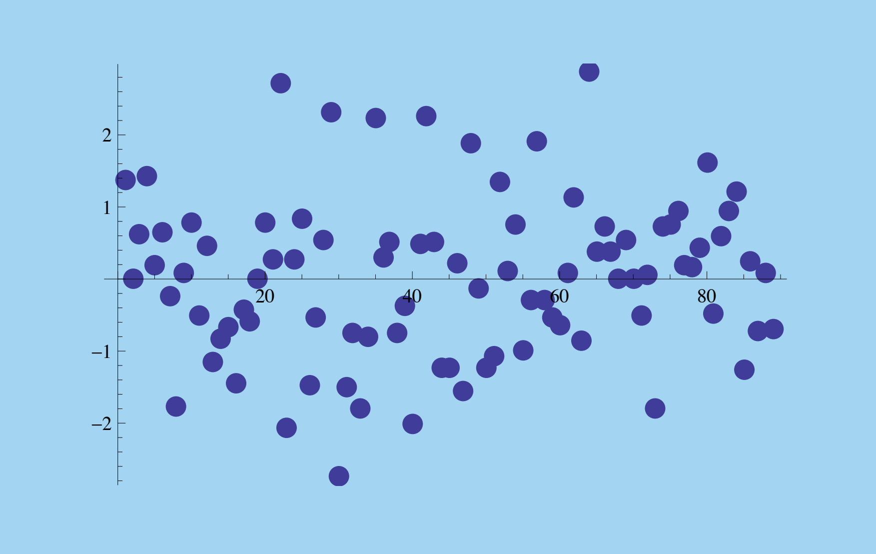
Porco et al. 2007, Sremcevic et al. 2007, Tiscareno et al. 2006

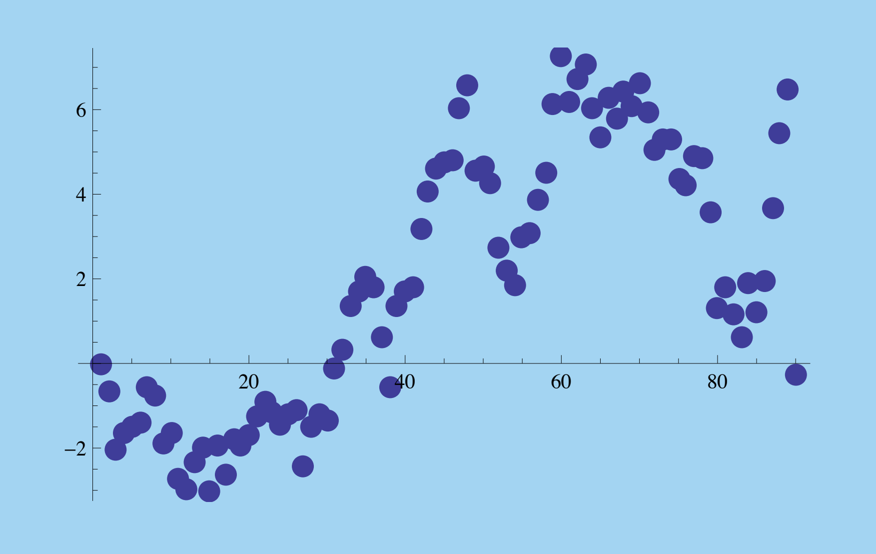
Observational evidence of non-Keplerian motion

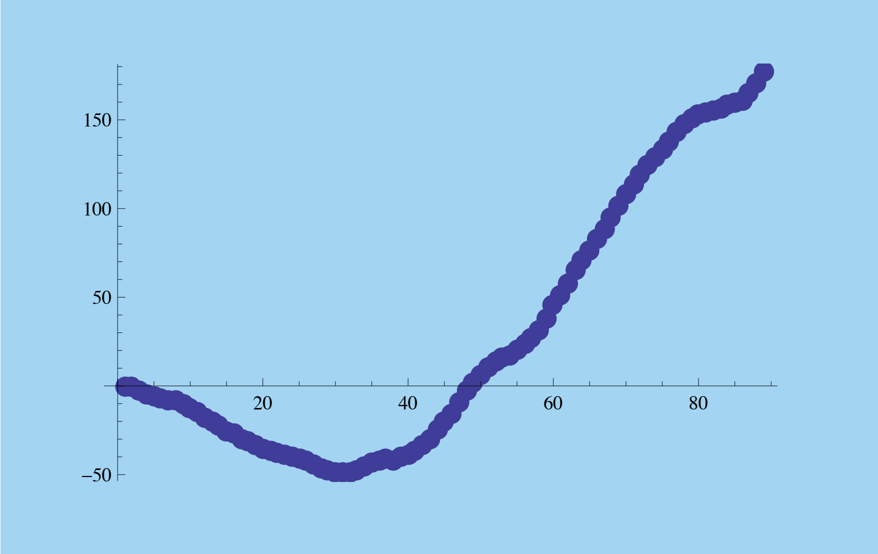


Tiscareno et al. 2010

Integrated random walk







Noise

Random walk

Integrated random walk

$$\xi_i$$

$$a_i = \sum_{j<i} \xi_j$$

$$\lambda_i = \sum_{j<i} a_j = \sum_{j<i} \sum_{k<j} \xi_k$$

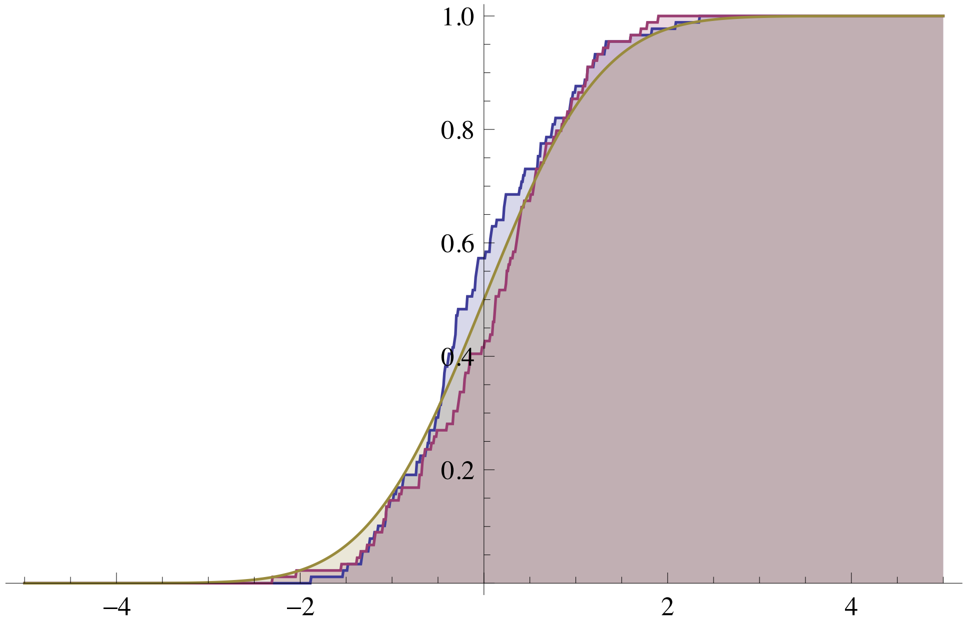
Work in progress: a statistical measure

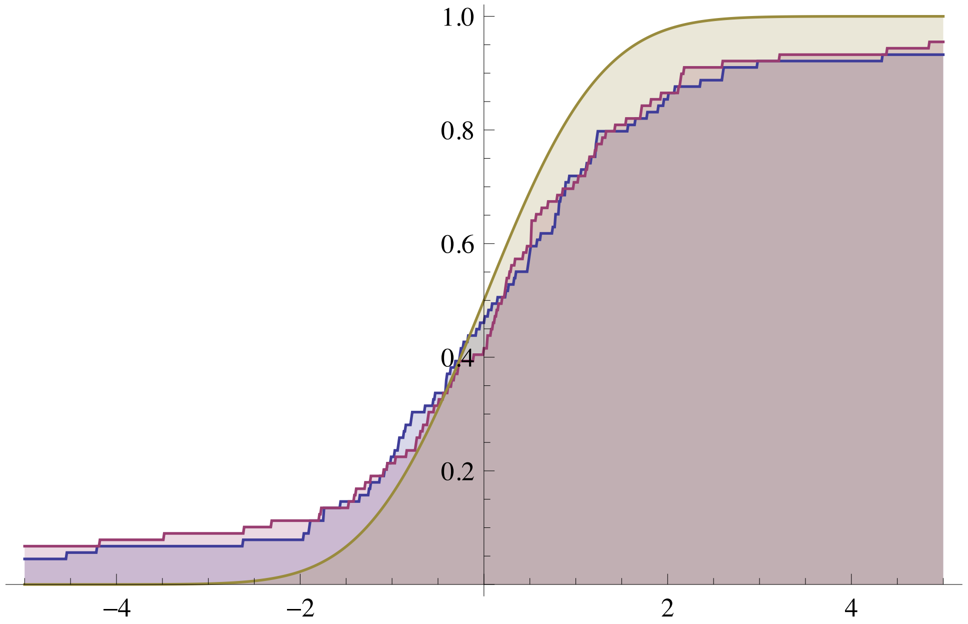
Create covariance matrix for the longitude residual assuming a Gaussian random walk

Find basis in which covariance matrix is diagonal

Project observation of longitude residuals to this basis

Compare distribution with normal distribution





Random walk

Analytic model

Describing evolution in a statistical manner

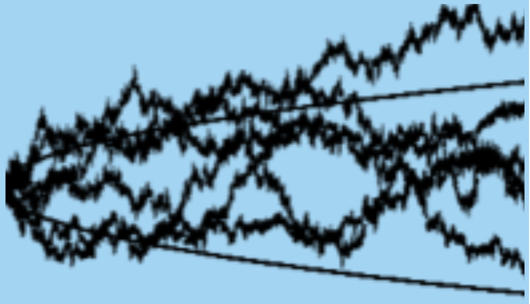
Partly based on Rein & Papaloizou 2009

$$\begin{aligned}\Delta a &= \sqrt{4\frac{Dt}{n^2}}\\ \Delta e &= \sqrt{2.5\frac{\gamma Dt}{n^2a^2}}\end{aligned}$$

N-body simulations

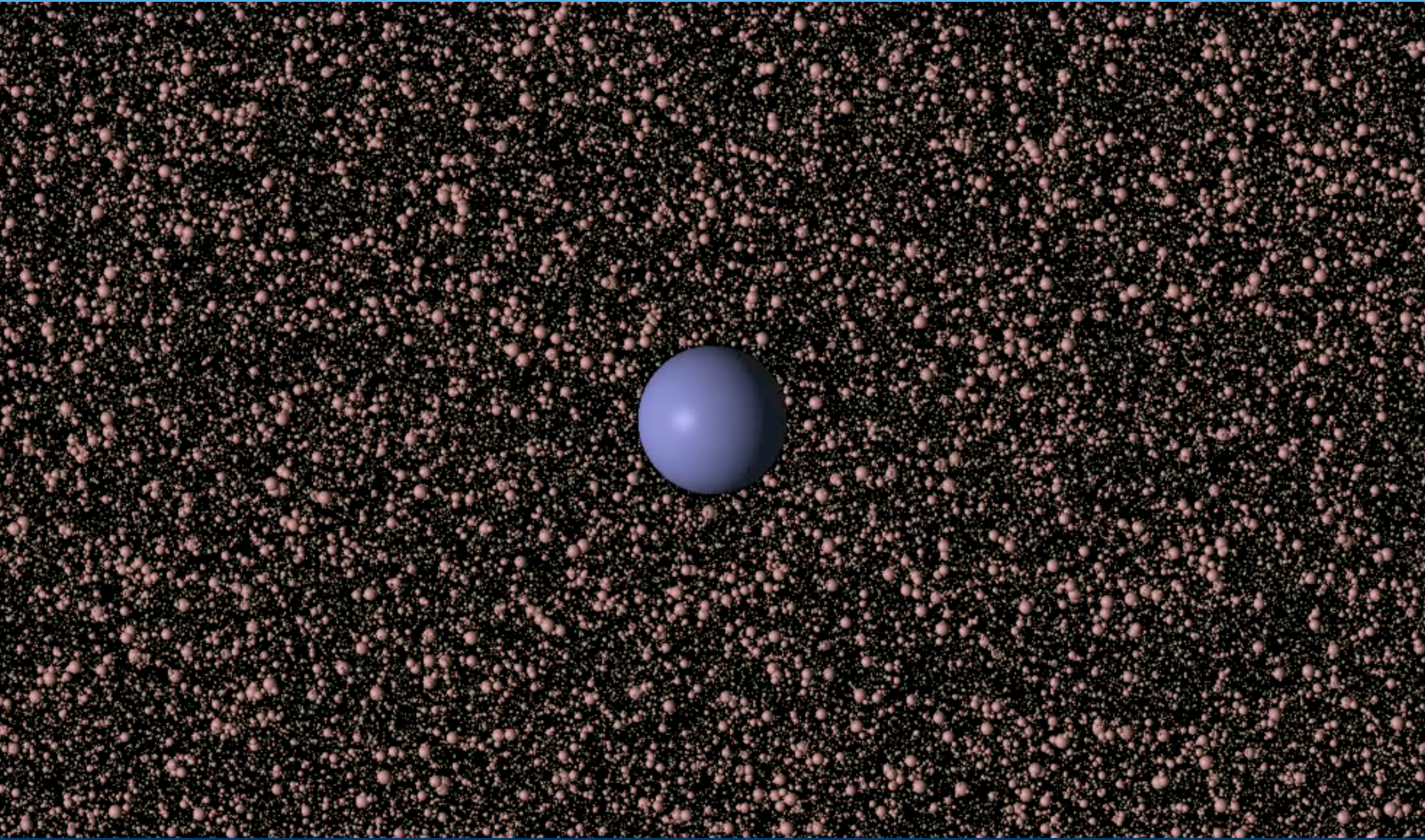
Measuring random forces or integrating moonlet directly

Crida et al 2010, Rein & Papaloizou 2010



Rein & Papaloizou 2010, Crida et al 2010

Random walk



REBOUND code, Rein & Papaloizou 2010, Crida et al 2010

Take home message V

Saturn's rings

=

small scale version of
a proto-planetary disc

REBOUND

A new open source collisional N-body code

REBOUND

- Multi-purpose N-body code
- Optimized for collisional dynamics
- Code description paper recently accepted by A&A
- Written in C, open source
- Freely available at http://github.com/hannorein/rebound

Astronomy & Astrophysics manuscript no. paper
October 20, 2011

REBOUND: An open-source multi-purpose N-body code for collisional dynamics

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e-mail: `liushangfei@pku.edu.cn`

Submitted: 13 September 2011

ABSTRACT

REBOUND is a new multi-purpose N-body code which is freely available under an open-source license. It was designed for collisional dynamics such as planetary rings but can also solve the classical N-body problem. It is highly modular and can be customized easily to work on a wide variety of different problems in astrophysics and beyond.

REBOUND comes with three symplectic integrators: leap-frog, the symplectic epicycle integrator (SEI) and a Wisdom-Holman mapping (WH). It supports open, periodic and shearing-sheet boundary conditions. REBOUND can use a Barnes-Hut tree to calculate both self-gravity and collisions. These modules are fully parallelized with MPI as well as OpenMP. The former makes use of a static domain decomposition and a distributed essential tree. Two new collision detection modules based on a plane-sweep algorithm are also implemented. The performance of the plane-sweep algorithm is superior to a tree code for simulations in which one dimension is much longer than the other two and in simulations which are quasi-two dimensional with less than one million particles.

In this work, we discuss the different algorithms implemented in REBOUND, the philosophy behind the code's structure as well as implementation specific details of the different modules. We present results of accuracy and scaling tests which show that the code can run efficiently on both desktop machines and large computing clusters.

Key words. Methods: numerical – Planets and satellites: rings – Proto-planetary disks

1. Introduction

REBOUND is a new open-source collisional N-body code. This code, and precursors of it, have already been used in wide variety of publications (Rein & Papaloizou 2010; Crida et al. 2010; Rein et al. 2010, Rein & Liu in preparation; Rein & Latter in preparation). We believe that REBOUND can be of great use for many different problems and have a wide reach in astrophysics and other disciplines. To our knowledge, there is currently no publicly available code for collisional dynamics capable of solving the problems described in this paper. This is why we decided to make it freely available under the open-source license GPLv3[1].

Collisional N-body simulations are extensively used in astrophysics. A classical application is a planetary ring (see e.g. Wisdom & Tremaine 1988; Salo 1991; Richardson 1994; Lewis & Stewart 2009; Rein & Papaloizou 2010; Michikoshi & Kokubo 2011, and references therein) which have often a collision time-scale that is much shorter than or at least comparable to an orbital time-scale. Self-gravity plays an important role, especially in the dense parts of Saturn's rings (Schmidt et al. 2009). These simulations are usually done in the shearing sheet approximation (Hill 1878).

Collisions are also important during planetesimal formation (Johansen et al. 2007; Rein et al. 2010, Johansen et al. in preparation). Collisions provide the dissipative mechanism to form a planetesimal out of a gravitationally bound swarm of boulders.

REBOUND can also be used with little modification in situa-
tions where only a statistical measure of the collision frequency
is required such as in transitional and debris discs. In such sy
tems, individual collisions between particles are not modeled e
actly, but approximated by the use of super-particles (Star
Kuchner 2009; Lithwick & Chiang 2007).

Furthermore, REBOUND can be used to simulate classi
body problems involving entirely collision-less systems.
plectic and mixed variable integrator can be used to fol
trajectories of both test-particles and massive particles.

We describe the general structure of the code, h
tain, compile and run it in Sect. 2. The time-steppi
and our implementation of symplectic integrators a
in Sect. 3. The modules for gravity are described in
algorithms for collision detection are discussed
Sect. 6, we present results of accuracy tests for
ules. We discuss the efficiency of the parallelizati
of scaling tests in Sect. 7. We finally summariz

2. Overview of the code structure

REBOUND is written entirely in C and conf
standard. It compiles and runs on any mode
which supports the POSIX standard suc
Mac OSX. In its simplest form, REBOU
libraries to compile.

Users are encouraged to install th
braries which enable real-time and in
LIBPNG is required to automatica

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[1] The full license is distributed together with REBOUND. It can also be downloaded from `http://www.gnu.org/licenses/gpl.html`.

REBOUND modules

Geometry

- Open boundary conditions
- Periodic boundary conditions
- Shearing sheet / Hill's approximation

Integrators

- Leap frog
- Symplectic Epicycle integrator (SEI)
- Wisdom-Holman mapping (WH)

Gravity

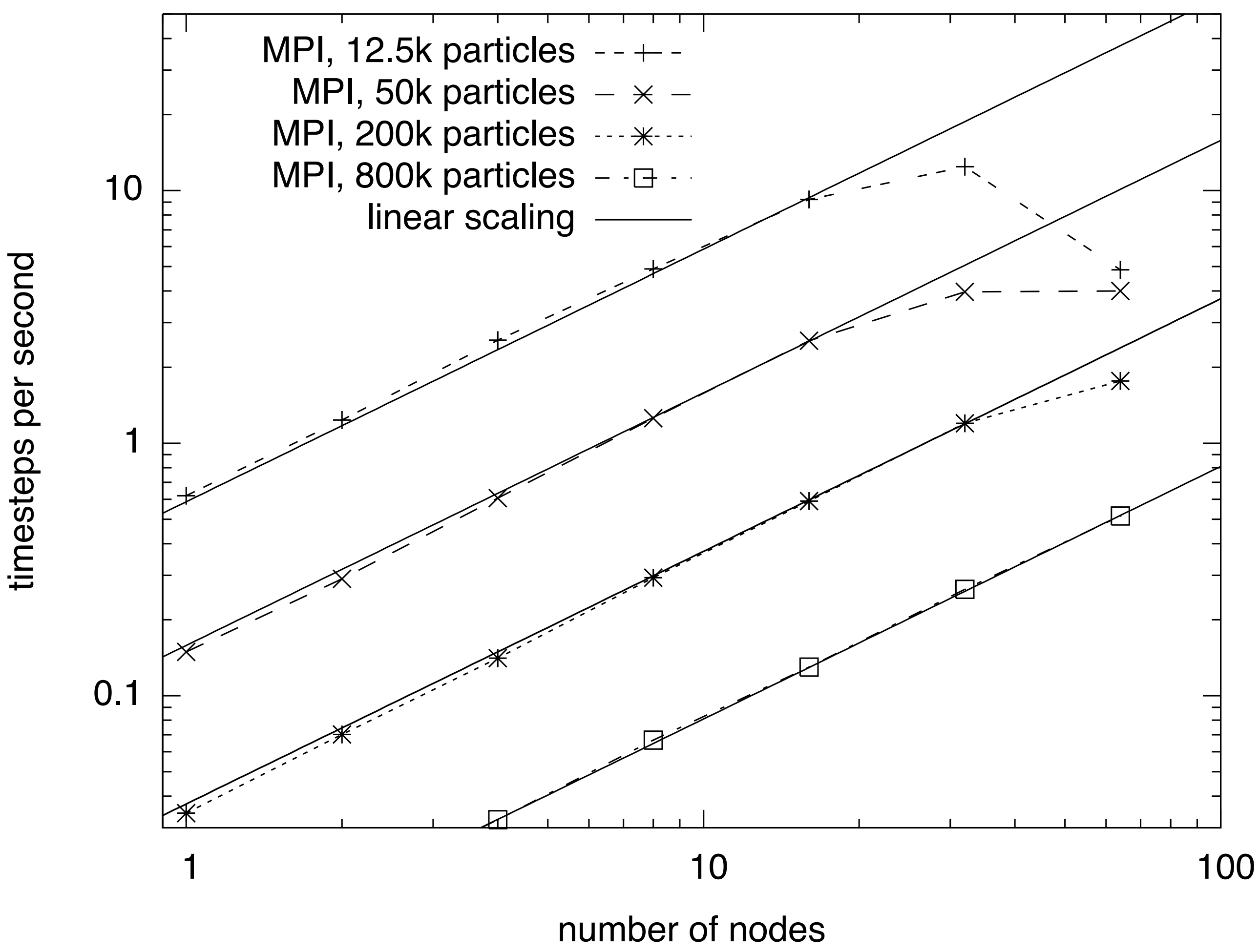
- Direct summation, $O(N^2)$
- BH-Tree code, $O(N \log(N))$
- FFT method, $O(N \log(N))$

Collision detection

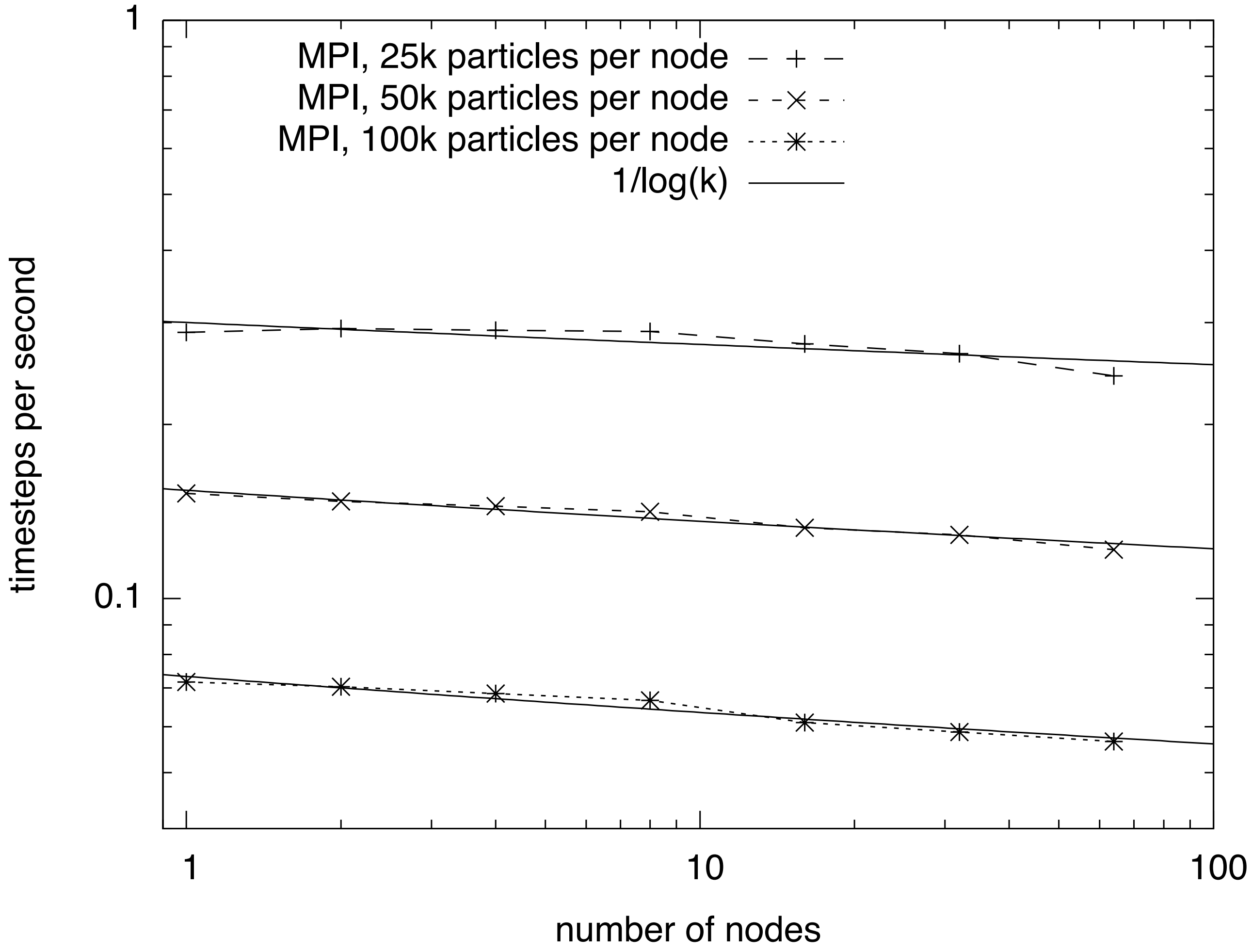
- Direct nearest neighbor search, $O(N^2)$
- BH-Tree code, $O(N \log(N))$
- Plane sweep algorithm, $O(N)$ or $O(N^2)$

REBOUND scalings using a tree

strong



weak



REBOUND

DEMO

Take home message VII

Download REBOUND

Conclusions

Conclusions

Resonances and multi-planetary systems

Multi-planetary system provide insight in otherwise unobservable formation phase

GJ876	formed in the presence of a disc and dissipative forces
HD128311	formed in a turbulent disc
HD45364	formed in a massive disc
HD200964	did not form at all

Moonlets in Saturn's rings

Small scale version of the proto-planetary disc
Random walk can be directly observed
Caused by collisions and gravitational wakes

REBOUND

N-body code, optimized for collisional dynamics, uses symplectic integrators
Open source, freely available, modular and easy to use
http://github.com/hannorein/rebound